Trigonometry - Functions and Graphs Lesson #9:
Sinusoidal Functions

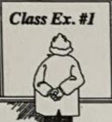
Sinusoidal Functions

A function whose graph resembles the sine or cosine curve is called a **sinusoidal function.** The graph of a sinusoidal function is called a sinusoidal graph. Many periodic phenomena have sinusoidal graphs, e.g. the time of sunrise as a function of the day of the year, the height of a chair of a ferris wheel as a function of time, the depth of the ocean due to changing tides as a function of time, etc.

In this lesson the equation of the sinusoidal function will be given. In the next lesson we will derive the equation of the sinusoidal function from given information.

Most of the equations used will be functions of time and the variable used will be t. The period of the graph will be in time units. Graphical methods will be used to solve problems and determining a suitable window is an essential feature of the solution.

Class Ex. #1

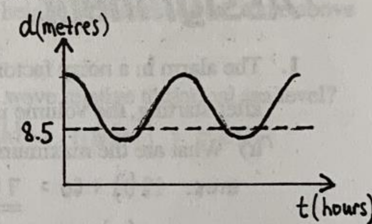


The depth, d metres, of water in a harbour, t hours after midnight, can be approximated by the function $d(t) = 12 + 5\cos 0.5t$, where $0 \leq t \leq 24$.

a) Determine the maximum and minimum depths of water in the harbour.

max: $12 + 5(1) = 17$ m

min: $12 + 5(-1) = 7$ m



b) Determine the period of the function.

period $= \dfrac{2\pi}{0.5} = 4\pi$ hours

c) Write a suitable window which can be used to display the graph of the function.

x: [0, 24, 2]   y: [0, 20, 5]   Answers may vary.

d) What is the depth of water, to the nearest tenth of a metre at 2:00 a.m.?

$t = 2$ 14.7 m

e) A ship which requires a minimum of 8.5 metres of water is in harbour at midnight. By what time, to the nearest minute, must it leave to prevent grounding?

graph $y_1 = 12 + 5\cos 0.5x$ Intersect at $x = 4.6923876$ $0.6923876 \times 60 = 41.54...$ (round down)

$y_2 = 8.5$

Must leave by 4:41 a.m

f) What is the next time, to the nearest necessary minute, that the ship can return to the harbour?

Intersect at $x = 7.873983$

$0.873983 \times 60 = 52.43...$ (round up)

return 7:53 am

Class Ex. #2

In a certain town in Alberta, the time of sunrise for any day can be found using the formula

$$t = -1.79 \sin\left(2\pi\frac{(d-78)}{365}\right) + 6.3$$

where t is the time in hours after midnight and d is the number of the day in the year.

a) Write a suitable window which can be used to display the graph of the function.

max $t = -1.79(-1) + 6.3 = 8.09$ $\quad x: [0, 365, 30] \quad y: [2, 10, 2]$

min $t = -1.79(1) + 6.3 = 4.51$

answers may vary

b) Use the formula to determine, to the nearest minute, when the sun rose on May 7, the 127th day of the year.

$x = 127 \quad y = 4.96292 \quad \underline{\underline{4:58 \text{ am}}}$

$0.96292 \times 60 = 58$ (nearest min.)

c) Determine on which days of the year the sun rose at 7 a.m.

graph $y_2 = 7$ $\quad$ intersect $\begin{cases} 54.659\ldots \\ 283.840\ldots \end{cases}$

$\underline{\underline{\text{day } 55 \text{ and day } 284}}$