Trigonometry - Equations and Identities Lesson #1:
Solving First Degree Trigonometric Equations

Overview

In this unit, we will

- solve, algebraically and graphically, first and second degree trigonometric equations expressed in degrees and radians, with
  i) a restricted domain
  ii) an unrestricted domain leading to a general solution
- prove trigonometric identities using reciprocal identities, quotient identities, Pythagorean identities, sum or difference identities, and double angle identities.

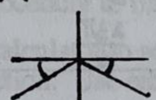
Review

Use an algebraic procedure to solve the following equations on the given domain.

a) $\sin x = -\frac{1}{2}, \quad 0 \le x \le 2\pi.$

quadrants 3/4

ref. $\angle = \frac{\pi}{6}$



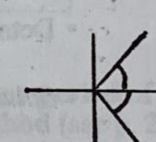
$x = \pi + \frac{\pi}{6}, \ 2\pi - \frac{\pi}{6}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

b) $3\sec x - 5 = 0, \ 0° \le x \le 360°$, to the nearest degree

$\sec x = \frac{5}{3} \quad \cos x = \frac{3}{5}$

quadrants 1/4



ref. $\angle = 53°$

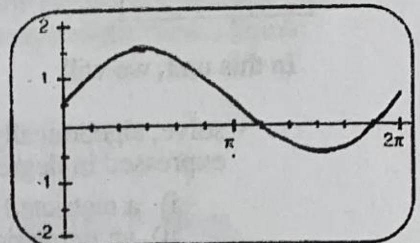
$x = 53°, \ 360 - 53°$

$x = 53°, 307°$

General Solution

The **general solution** to a trigonometric equation is the solution over the **domain of real numbers**. We will investigate how to determine a general solution graphically and algebraically in this lesson.

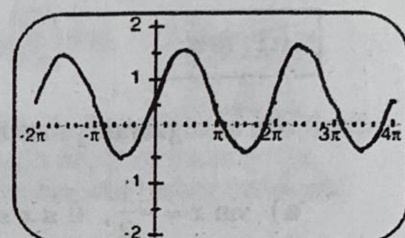
Exploring a General Solution Using a Graphical Approach



Consider the equation $\sin x = -\frac{1}{2}$ (i.e. $\sin x + \frac{1}{2} = 0$).

a) Use the following graphical method to estimate the solution to the equation on the domain $0 \le x \le 2\pi$.

- Use window x: $[0, 2\pi, \frac{\pi}{6}]$ y:$[-2, 2, 0.5]$.
- Graph $Y_1 = \sin x + \frac{1}{2}$.
- Determine (in terms of π), the x-intercepts of graph Y_1 where $0 \le x \le 2\pi$.
- *Solution is* $\frac{7\pi}{6}, \frac{11\pi}{6}$



b) Use the following graphical method to estimate the solution to the equation on the domain $-2\pi \le x \le 4\pi$.

- Use the window x: $[-2\pi, , 4\pi, \frac{\pi}{6}]$ y:$[-2, 2, 0.5]$.
- Graph $Y_1 = \sin x + \frac{1}{2}$.
- Determine, as exact values, the x-intercepts of graph Y_1 where $-2\pi \le x \le 4\pi$.
- *Solution is* $-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$

c) Notice that the solution in b) forms two sets of answers which differ by 2π (or a multiple of 2π) from the answers in a). Complete:

Set 1 $\left(\text{derived from } \frac{7\pi}{6}\right)$: $-\frac{5\pi}{6}, \frac{7\pi}{6}, \frac{19\pi}{6}$ are angles which differ by 2π (or a multiple of 2π).

Set 2 $\left(\text{derived from } \frac{11\pi}{6}\right)$: $-\frac{\pi}{6}, \frac{11\pi}{6}, \frac{23\pi}{6}$ are angles which differ by 2π (or a multiple of 2π).

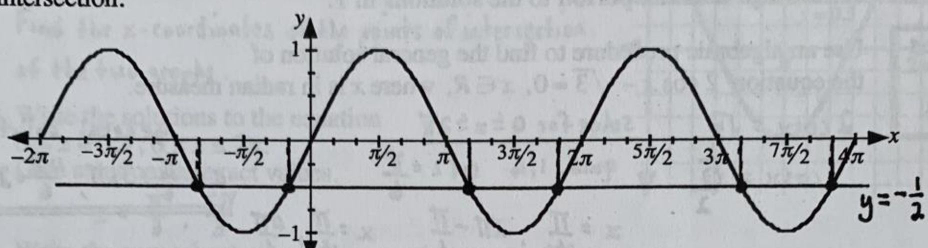
d) If the domain is changed to $-4\pi \le x \le 6\pi$, write two more solutions for each set.

Set 1 : $-\frac{17\pi}{6}, \frac{31\pi}{6}$ Set 2 : $-\frac{13\pi}{6}, \frac{35\pi}{6}$

e) Use the ideas above to write the general solution to the equation $\sin x = -\frac{1}{2}$ where the domain is unrestricted, i.e. $x \in R$.

- *General Solution is* $x = \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi, n \in I$

f) The solution to the equation $\sin x = -\frac{1}{2}$ in part b) may also be obtained by using the intersection of two graphs. Show the solution to the equation $\sin x = -\frac{1}{2}$ on the given domain by drawing the line with equation $y = -\frac{1}{2}$ and marking the points of intersection.

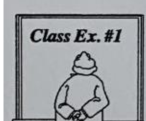


The answers in parts b), c) and d) differ by 2π radians because the graph of $y = \sin x$ has a **period** of 2π radians.

Determining a General Solution Using a Graphical Approach

Use the following procedure to find the general solution.

1. Use a graphing calculator to solve the equation where the domain is **one period** of the graph of the function. Use either the **Intersect Method** or **Zero Method** (see p. 235).
2. Determine the general solution by adding or subtracting **multiples of the period** of the graph of the function to the solutions in 1.



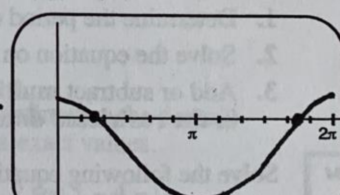
Solve the equation $\cos x - 0.75 = 0,\ x \in R$, using two different graphical approaches. Give answers to the nearest hundredth of a radian.

graph $y = \cos x - 0.75$ on domain $0 \leq x \leq 2\pi$

determine the x-intercepts using the zero feature.

0.72, 5.56

general solution $x = 0.72 + 2n\pi,\ 5.56 + 2n\pi,\ n \in I$

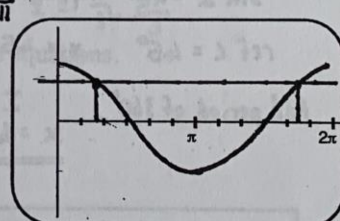


graph $y_1 = \cos x$ and $y_2 = 0.75$ on domain $0 \leq x \leq 2\pi$

determine the x-coordinates of the points of intersection using the intersect feature.

0.72, 5.56

general solution $x = 0.72 + 2n\pi,\ 5.56 + 2n\pi,\ n \in I$

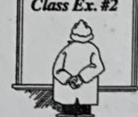


General Solution Using an Algebraic Approach

Use the following procedure to find the general solution using an algebraic approach.

1. Solve the equation where the domain is **one period** of the graph of the function.
2. The general solution can be determined by adding or subtracting **multiples of the period** to the solutions in 1.

Class Ex. #2



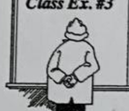
Use an algebraic procedure to find the general solution of the equation $2\cos x - \sqrt{3} = 0$, $x \in R$, where x is in radian measure.

$2\cos x = \sqrt{3}$ Solve for $0 \le x \le 2\pi$ General solution

$\cos x = \frac{\sqrt{3}}{2}$ quad. 1/4   ref ∠ $= \frac{\pi}{6}$ $x = \frac{\pi}{6} + 2n\pi,\ \frac{11\pi}{6} + 2n\pi,\ n \in I$

$x = \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$ $x = \frac{\pi}{6},\ \frac{11\pi}{6}$

Class Ex. #3



In some cases, the different parts of a general solution can be combined together in one. Determine the general solution, in radians, of the equation

a) $\sin x = 0$

quadrants 1 - 4   ref. ∠ $= 0$

on $0 \le x \le 2\pi$, $x = 0, \pi, 2\pi$

general solution $x = 2n\pi,\ \pi + 2n\pi,\ 2\pi + 2n\pi,\ n \in I$

combine to $x = n\pi,\ n \in I$

b) $\cos x = 0$

quadrants 1-4   ref. ∠ $= \frac{\pi}{2}$

on $0 \le x \le 2\pi$, $x = \frac{\pi}{2}, \frac{3\pi}{2}$

general solution $x = \frac{\pi}{2} + 2n\pi,\ \frac{3\pi}{2} + 2n\pi,\ n \in I$

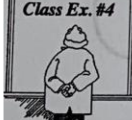
combine to $x = \frac{\pi}{2} + n\pi,\ n \in I$

Solving on a Restricted Domain

Use the following procedure to solve a trigonometric equation on a restricted domain.

1. Determine the period of the trigonometric function.
2. Solve the equation on the domain $0 \le x \le$ period.
3. Add or subtract **multiples of the period** to the solutions in 1 to solve **in the restricted domain.**

Class Ex. #4



Solve the following equations on the specified domain.

a) $2\sin x - \sqrt{2} = 0$ for $360° \le x \le 720°$

$\sin x = \frac{\sqrt{2}}{2}$, quadrants 1/2

ref ∠ $= 45°$ $x = 45°, 135°$ on $0° \le x \le 360°$

Add period of 360°.

$x = 405°, 495°$

b) $\sqrt{3}\cot x + 1 = 0$ for $-\pi \le x \le 0$

$\cot x = \frac{-1}{\sqrt{3}}$, $\tan x = -\sqrt{3}$, quadrant 2

ref. ∠ $= \frac{\pi}{3}$ $x = \frac{2\pi}{3}$ on $0 \le x \le \pi$

Subtract period of π

$x = -\frac{\pi}{3}$

Complete Assignment Questions #1 - #15