Trigonometry - Equations and Identities Lesson #2:
Solving Second Degree Trigonometric Equations

In this lesson we will be solving **second degree** equations where the power of the trigonometric function is two (e.g. $\sin^2 x - 3\sin x = 0$).

Trigonometric equations which can be solved by using identities will be covered in lesson 6.

Factoring Trigonometric Expressions

Just as with polynomial expressions, trigonometric expressions can be factored. The ability to factor trigonometric expressions is a useful skill in two areas:

- solving trigonometric equations (in this lesson)
- proving trigonometric identities (in lesson 6)

In factoring trigonometric expressions we can apply three basic factoring techniques - common factor, difference of two squares, and trinomials of the form $ax^2 + bx + c, a \neq 0$.

Class Ex. #1

Factor the following trigonometric expressions:

a) $8\tan A + 4$

$= 4(2\tan A + 1)$

b) $\sin^2 x - 3\sin x$

$= \sin x(\sin x - 3)$

c) $4\sin^2 x - 1$

$= (2\sin x - 1)(2\sin x + 1)$

d) $\csc^2 x - 3\csc x - 28$

$= (\csc x - 7)(\csc x + 4)$

e) $2\cos^2 x + 7\cos x - 4$

$= 2\cos^2 x - \cos x + 8\cos x - 4$

$= \cos x(2\cos x - 1) + 4(2\cos x - 1)$

$= (2\cos x - 1)(\cos x + 4)$

Complete Assignment Question #1

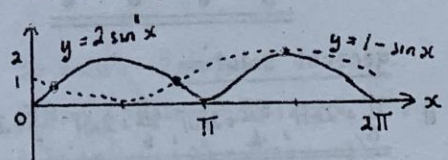
Solving a Second Degree Equation Using a Graphical Approach

Class Ex. #2

Consider the equation $2\sin^2 x = 1 - \sin x$.

a) Use a **graphical** approach to determine the roots of the equation where $0 \leq x \leq 2\pi$. Give solutions as exact multiples of π.

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$



b) State the general solution to the equation.

$x = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{3\pi}{2} + 2n\pi, n \in I$

Solving a Second Degree Equation Using an Algebraic Approach

Class Ex. #3

Consider the equation $2\sin^2 x = 1 - \sin x$.

a) Use an **algebraic** approach to determine the roots of the equation where $0 \le x \le 2\pi$. Give solutions as exact values.

$2\sin^2 x + \sin x - 1 = 0$

$2\sin^2 x - \sin x + 2\sin x - 1 = 0$

$\sin x(2\sin x - 1) + 1(2\sin x - 1) = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

$\sin x = \frac{1}{2}$
quadrant 1/2
ref. $\angle = \frac{\pi}{6}$
$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$
$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin x = -1$
quadrant 3/4
ref. $\angle = \frac{\pi}{2}$
$x = \pi + \frac{\pi}{2}, 2\pi - \frac{\pi}{2}$
$x = \frac{3\pi}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

b) State the general solution to the equation.

$x = \frac{\pi}{6} + 2n\pi,\ \frac{5\pi}{6} + 2n\pi,\ \frac{3\pi}{2} + 2n\pi,\ n \in I$

Class Ex. #4

Use an algebraic procedure to determine the roots of the equation on the given domain and write the general solution.

a) $4\sin^2 A - 1 = 0, 0 \le A \le 2\pi$

$(2\sin A - 1)(2\sin A + 1) = 0$

$\sin A = \frac{1}{2}$ or $\sin A = -\frac{1}{2}$

quad. 1/2
ref. $\angle = \frac{\pi}{6}$
$A = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

quad. 3/4
ref. $\angle = \frac{\pi}{6}$
$A = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$A = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

general solution

$A = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi, n \in I$

b) $\tan^2 x + \tan x = 0,\ 0 \le x \le 2\pi$

$\tan x(\tan x + 1) = 0$

$\tan x = 0$ or $\tan x = -1$

$x = 0, \pi, 2\pi$

quad 2/4
ref. $\angle = \frac{\pi}{4}$
$x = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$
$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$x = 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi$

general solution

$x = n\pi, \frac{3\pi}{4} + n\pi, n \in I$

Class Ex. #5

Determine the zeros of the following functions on the specified domain.

a) $f(x) = \csc^2 x - 3\csc x - 28$, domain $0° \leq x \leq 180°$

Answer to the nearest degree

$(\csc x - 7)(\csc x + 4) = 0$

$\csc x = 7$ or $\csc x = -4$

$\sin x = \frac{1}{7}$ or $\sin x = -\frac{1}{4}$

quad. 1/2 — no solution on domain

ref. $\angle = 8°$

$x = 8°, 180° - 8°$

$\underline{\underline{x = 8°, 172°}}$

$\underline{\underline{\text{zeros are } 8° \text{ and } 172°}}$

b) $g(\theta) = 2\cos^2\theta + 5\cos\theta - 3$, domain $-\pi \leq \theta \leq \pi$

$2\cos^2\theta - \cos\theta + 6\cos\theta - 3$

$= \cos\theta(2\cos\theta - 1) + 3(2\cos\theta - 1)$

$= (2\cos\theta - 1)(\cos\theta + 3)$

$g(\theta) = 0 \Rightarrow \cos\theta = \frac{1}{2}$ or $\cos\theta = -3$

quad 1/4 ref. $\angle = \frac{\pi}{3}$ — no solution

on $0 \leq \theta \leq 2\pi$, $\theta = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{11\pi}{3}$

on $-\pi \leq \theta \leq \pi$, $\theta = \frac{\pi}{3}, \frac{11\pi}{3} - 2\pi$

$\theta = \frac{\pi}{3}, \frac{-\pi}{3}$

$\underline{\underline{\text{zeros are } \frac{\pi}{3} \text{ and } -\frac{\pi}{3}}}$

Complete Assignment Questions #2 - #13