Absolute Value Functions and Reciprocal Functions Lesson #2: Solving Absolute Value Equations - Part One

Absolute Value Equations

$|2x+3| = 8$, $|3+x| = 2x+1$, $|2x-3| - |x+4| = 8$, and $|x^2 - 17| = 8$ are all examples of **absolute value equations**.

In the next two lessons, we will learn how to solve these equations graphically and algebraically.

Solving Absolute Value Equations Using a Graphing Calculator

Intersection Method

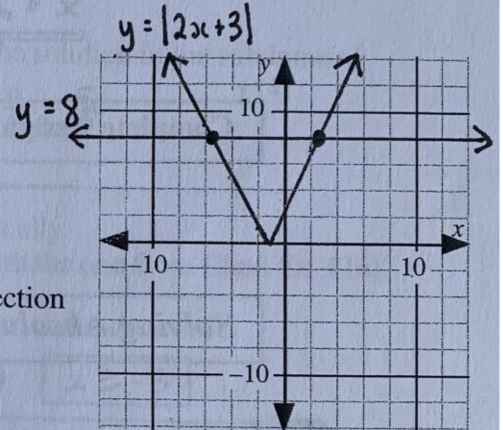
To solve the equation $|2x+3| = 8$ by the intersection method, use the following procedure.

1. Graph $Y_1 = |2x+3|$.

2. Graph $Y_2 = 8$.

3. Find the x-coordinate(s) of the point(s) of intersection using the `intersect` feature of the calculator.

$x = -\frac{11}{2}, \frac{5}{2}$



x-intercept Method

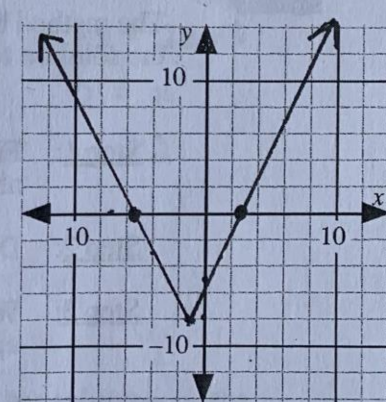
To solve the equation $|2x+3| = 8$ by the zero method, use the following procedure.

1. Rearrange the original equation with all terms on the left hand side and 0 on the right side to get $|2x+3| - 8 = 0$.

2. Graph $Y_1 = |2x+3| - 8$.

3. Use the `zero` feature of the calculator to find the x-intercept(s).

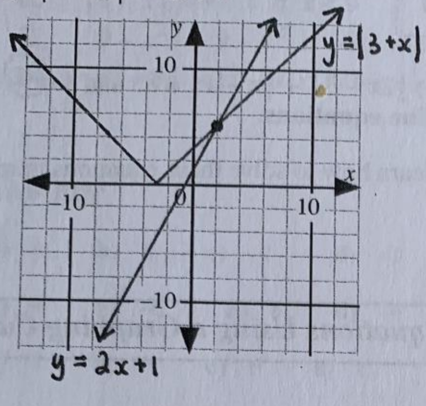
$x = -\frac{11}{2}, \frac{5}{2}$



Class Ex. #1

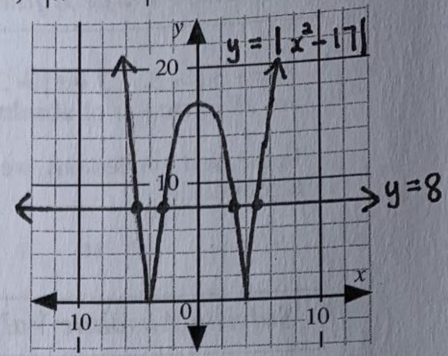
Solve the following absolute value equations by graphing.

a) $|3 + x| = 2x + 1$



$x = 2$

b) $|x^2 - 17| = 8$



$x = \pm 3, \pm 5$

Complete Assignment Question #1 - #3

Solving Absolute Value Equations Algebraically

Note

There are different ways in which to determine algebraically the solution to absolute value equations.

The method below has the advantage that a virtually identical method can be used to determine the solution to absolute value inequalities in calculus and other higher level math courses.

Step 1: Find the value(s) of the variable which will make the expression within the absolute value symbol(s) equal to zero.

Step 2: Divide the domain into smaller subdomains using the value(s) found in Step 1.

Step 3: Write the absolute value expressions in piecewise form, using the piece that is appropriate for each subdomain.

Step 4: Solve the resulting equation in each subdomain.

Step 5: Check that the solution to each equation is in the subdomain, and combine all valid solutions.

Class Ex. #2

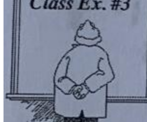
Alan has started to solve the equation $|2x+3| = 8$ using the steps on the previous page. Complete Alan's work and check the solution with the graphical solution on page 457.

Alan's solution

	subdomain $x < -\frac{3}{2}$	$-\frac{3}{2}$	subdomain $x \geq -\frac{3}{2}$
number line	◀—————————	+	—————————▶

Solve $\lvert 2x+3 \rvert = 8$	Solve $\lvert 2x+3 \rvert = 8$
$-(2x+3) = 8$	$2x + 3 = 8$
$-2x - 3 = 8$	$2x = 5$
$-2x = 11$	$x = \frac{5}{2}$
$x = -\frac{11}{2}$	
Is the solution in the subdomain? yes	Is the solution in the subdomain? yes

Final solution: $x = -\frac{11}{2}, \frac{5}{2}$



Class Ex. #3

Haley solved the equation $|3+x| = 2x+1$ algebraically. Complete Haley's work and compare the solution with the one from Class Ex. #1a).

Haley's solution

	subdomain $\boxed{x < -3}$	-3	subdomain $\boxed{x \geq -3}$
number line	◀—————————	+	—————————▶

solve $\lvert 3+x \rvert = 2x+1$	solve $\lvert 3+x \rvert = 2x+1$
$-3 - x = 2x + 1$	$3 + x = 2x + 1$
$-4 = 3x$	$2 = x$
$x = -\frac{4}{3}$	$x = 2$
Is the solution in the subdomain? no	Is the solution in the subdomain? yes

Final solution: $x =$ 2

Complete Assignment Question #4

An Alternative Method for Solving Single Absolute Value Equations

For equations containing a single absolute value expression, such as in Class Ex #2 and #3, and assignment question #4, there is an alternative method which can be used to determine the solution.

When we asked the question "Is the solution in the subdomain?", the answer was sometimes "yes" and sometimes "no". An alternative method is to solve the two pieces of the absolute value equation without considering the subdomains. **However, this means that the solution needs to be checked or verified to eliminate incorrect answers.**

Class Ex. #4

Consider the equation $|3 + x| = 2x + 1$.

a) Write $|3 + x|$ as a piecewise expression without using absolute value symbols.

$|3+x| = \begin{cases} 3+x, & \text{if } 3+x \geq 0 \\ -3-x, & \text{if } 3+x < 0 \end{cases}$ $\quad |3+x| = \begin{cases} 3+x, & x \geq -3 \\ -3-x, & x < -3 \end{cases}$

b) Set each of the expressions from a) equal to $2x + 1$, solve for x in each case, and verify.

$3+x = 2x+1$
$2 = x$
$x = 2$

verify $x = 2$
LS $= |3+2| = |5| = 5$
RS $= 2(2)+1 = 5$
LS $=$ RS

$-3-x = 2x+1$
$-4 = 3x$
$x = -\frac{4}{3}$
reject

verify $x = -\frac{4}{3}$
LS $= |3 + (-\frac{4}{3})| = |\frac{5}{3}| = \frac{5}{3}$
RS $= 2(-\frac{4}{3}) + 1 = -\frac{5}{3}$
LS $\neq$ RS

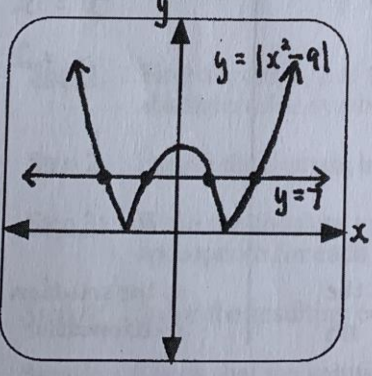
$x = 2$

c) Compare this method with the method in Class Ex #3. Same answer

Class Ex. #5

Consider the equation $|x^2 - 9| = 7$.

a) Solve the equation by graphing. Answer to two decimals places, if necessary.



$x = \pm 1.41, 4$

b) Solve the equation algebraically, using the method in Class Ex #4 b).

$x^2 - 9 = 7$
$x^2 = 16$
$x = \pm 4$

$-x^2 + 9 = 7$
$2 = x^2$
$x = \pm\sqrt{2}$

verify $x = -4$: LS $= |(-4)^2 - 9| = |7| = 7 =$ RS
verify $x = 4$: LS $= |(4)^2 - 9| = |7| = 7 =$ RS
verify $x = -\sqrt{2}$: LS $= |(-\sqrt{2})^2 - 9| = |-7| = 7 =$ RS
verify $x = \sqrt{2}$: LS $= |(\sqrt{2})^2 - 9| = |-7| = 7 =$ RS

$x = \pm\sqrt{2}, \pm 4$