Absolute Value Functions and Reciprocal Functions Lesson #1: Absolute Value Functions

The Absolute Value of a Number

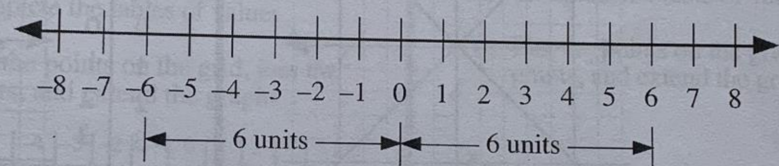
The **absolute value** of a real number can be defined as the principal square root of the square of the number.

e.g. the absolute value of $6 = \sqrt{(6)^2} = \sqrt{36} = 6$

the absolute value of $-6 = \sqrt{(-6)^2} = \sqrt{36} = 6$

For a real number, a, the absolute value of a is written $|a|$. e.g. $|6| = 6$ and $|-6| = 6$.

Note **The absolute value of a real number can be regarded as the distance of the number from zero on a number line.**



Note **The absolute value of a number will never be negative.**

Class Ex. #1

Evaluate:

a) $|3|$

3

b) $|-3|$

3

c) $-|8|$

-8

d) $-|-8|$

-8

e) $|-7| + |7|$

$7 + 7$

$= 14$

f) $|1 - 5|$

$|-4|$

$= 4$

g) $-|-\sqrt{81}\,|$

$-|-9|$

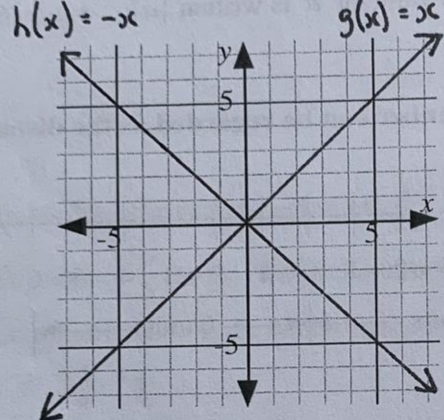
$= -9$

Complete Assignment Questions #1 - #3

Investigating the Function $f(x) = |x|$

1. Consider the functions $g(x) = x$, and $h(x) = -x$, whose graphs respectively have equations $y = x$ and $y = -x$.

Sketch and label each graph on the grid below.

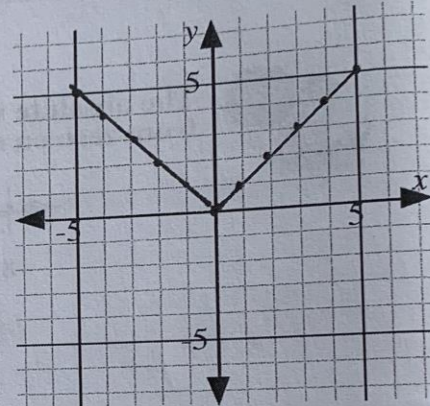


2. Consider the function $f(x) = |x|$, whose graph has the equation $y = |x|$.

a) Complete the table of values.

b) Plot the points on the grid and join the points.

x	y
–5	5
–4	4
–3	3
–2	2
–1	1
0	0
1	1
2	2
3	3
4	4
5	5



3. Explain the similarities and differences between the graphs on the two grids above.

when $y > 0$ the graph of $y = |x|$ is the same as the combined graph on the left.

when $y < 0$ the graph of $y = |x|$ has no points.

4. The graph of $y = |x|$ contains two straight lines.

a) State the equation of the line in quadrant 1. $y = x$

b) State the equation of the line in quadrant 2. $y = -x$

5. We can see from #4 that the equation $y = |x|$ can be written in two **pieces** with <u>different domains for each piece</u>.

Complete the following to write the absolute value function $f(x) = |x|$ as a **piecewise function**;

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Recall from the definition of the absolute value of a number on the previous page that the absolute value function $f(x) = |x|$ could be written as $f(x) = \sqrt{x^2}$.

Defining The Absolute Value Function $f(x) = |x|$

The absolute value function $f(x) = |x|$ can be defined as:

$$f(x) = |x| = \sqrt{x^2} = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Notice that when x is a positive number, $|x| = x$, and when x is a negative number, $|x| = -x$.

Investigating the Function $f(x) = |x - 3|$

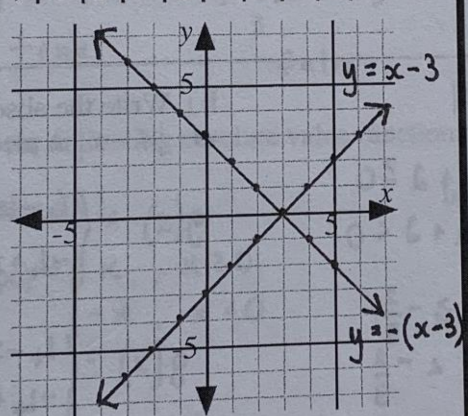
1. Consider the functions $g(x) = x - 3$, and $h(x) = -(x - 3)$, whose graphs respectively have equations $y = x - 3$ and $y = -(x - 3)$.

a) Complete the tables of values.

b) Plot the points on the grid, join the points, and extend the graphs.

x	−4	−3	−2	−1	0	1	2	3	4	5
$y = x - 3$	-7	-6	-5	-4	-3	-2	-1	0	1	2

x	−4	−3	−2	−1	0	1	2	3	4	5
$y = -(x - 3)$	7	6	5	4	3	2	1	0	-1	-2

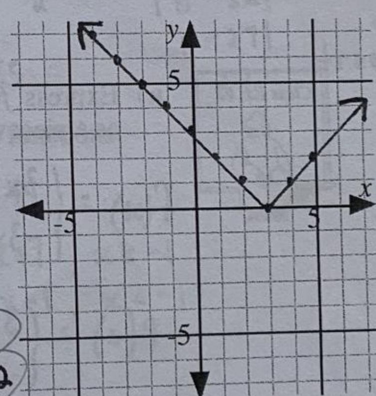


2. Consider the function $f(x) = |x - 3|$, whose graph has the equation $y = |x - 3|$.

a) Complete the table of values.

b) Plot the points on the grid, join the points, and extend the graph.

x	y
−5	8
−4	7
−3	6
−2	5
−1	4
0	3
1	2
2	1
3	0
4	-1
5	-2



3. The function $f(x) = |x - 3|$ is written below as a piecewise function, but the domain for each piece has been omitted.

Use the graph of $y = |x - 3|$ to determine the domain for each piece, and complete the piecewise function form of $f(x) = |x - 3|$ shown below.

$$f(x) = \begin{cases} x - 3 & \text{if } x \geq 3 \\ -(x - 3) & \text{if } x < 3 \end{cases}$$

4. Explain how the equation of the second piece can be determined from the equation of the first piece.

The equation of the second piece is the negative of the first piece.

5. Explain algebraically, without using a graph, how the domain for each piece could be determined.

The zero of the absolute value function, call it p, divides the absolute value function into two pieces with domains $x \geq p$ and $x < p$.

Note

- Every absolute value function can be defined in pieces.
- The absolute value of a quantity will always be the same quantity if the quantity is positive and the opposite quantity if the quantity is negative.
- Writing an absolute value function in piecewise form is an integral part of determining the solution to absolute value equations or inequalities.

Class Ex. #2

a) Express $f(x) = |3x + 2|$ as a piecewise function.

$$f(x) = \begin{cases} 3x+2, & 3x+2 \geq 0 \\ -3x-2, & 3x+2 < 0 \end{cases}$$

$$f(x) = \begin{cases} 3x+2, & x \geq -\frac{2}{3} \\ -3x-2, & x < -\frac{2}{3} \end{cases}$$

b) Write the absolute value expression $|4 - x|$ in piecewise form.

$$g(x) = \begin{cases} 4-x, & 4-x \geq 0 \\ -4+x, & 4-x < 0 \end{cases}$$

$$g(x) = \begin{cases} 4-x, & x \leq 4 \\ -4+x, & x > 4 \end{cases}$$

Complete Assignment Questions #4 - #11