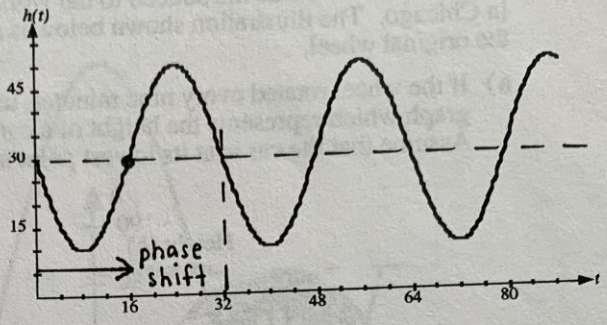
Trigonometry - Functions and Graphs Lesson #10:
Modelling Sinusoidal Functions

In the previous lesson we were asked to solve problems when we were given the equation of a sinusoidal function.

In this lesson we will derive the equation of the sinusoidal function from a graph.

Review

The sinusoidal wave shown has a maximum value of 50 and a minimum value of 10. Write the equation of the sinusoidal wave in the form $h(t) = a \sin [b(t - c)] + d$, where $a > 0$.



amplitude $= \frac{50-10}{2} = 20$ $\quad a = 20$

period $= 32$ $\quad b = \frac{2\pi}{32} = \frac{\pi}{16}$

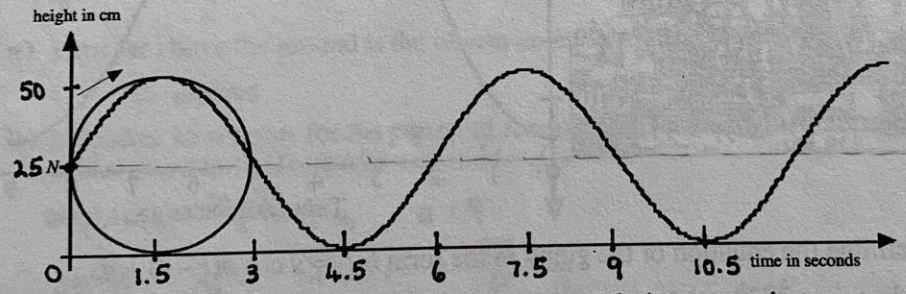
h.p.s $= 16$ right $\quad c = 16$

vert. disp. $= \frac{50+10}{2} = 30$ $\quad d = 30$

$$h(t) = 20 \sin \frac{\pi}{16}(t - 16) + 30$$

Class Ex. #1

A nail is caught in the tread of a rotating tire at point N in the following sketch.



The tire has a diameter of 50 cm and rotates at 10 revolutions per minute. After 4.5 seconds the nail touches the ground.

a) Use the information given to write a scale for each axis.

b) Determine the equation for the height of the nail as a function of time in the form $h(t) = a \sin bt + d$, where $a > 0$.

amplitude $= 25$cm $\quad a = 25$

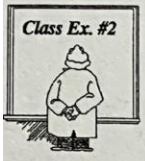
period $= 6$ sec. $\quad b = \frac{2\pi}{6} = \frac{\pi}{3}$

$$h(t) = 25 \sin \frac{\pi}{3} t + 25$$

vert. disp. $= 25$cm $\quad d = 25$

c) How far, to the nearest tenth of a centimetre, is the nail above the ground after 6.5 seconds?

$$h = 25 \sin\left(\frac{\pi}{3}(6.5)\right) + 25 = 37.5 \text{ cm}$$

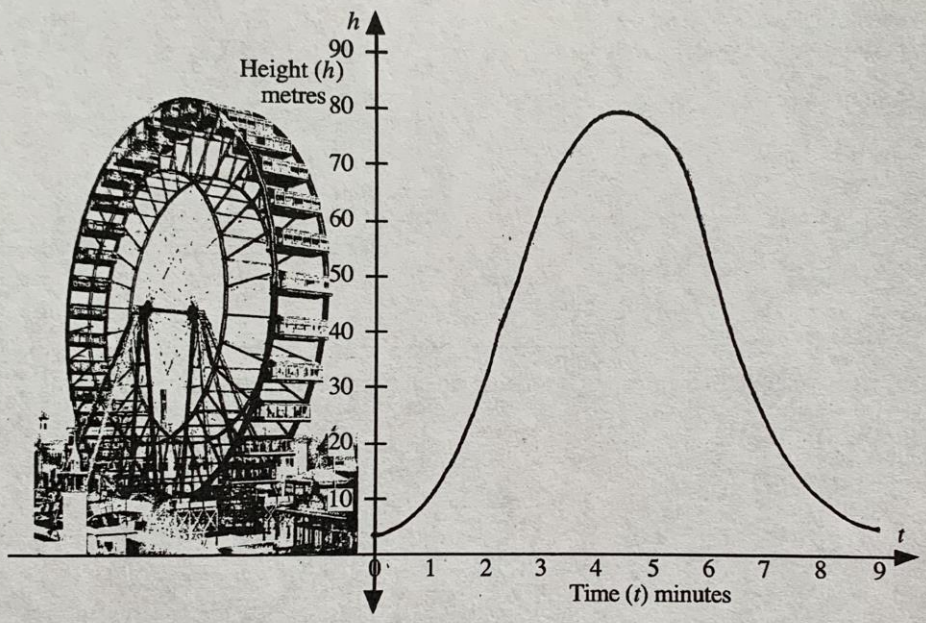


The first Ferris Wheel ever built was created by a bridge builder by the name of George W. Ferris in 1893. The diameter of the wheel was approximately 76 metres and the maximum height of the Ferris Wheel was approximately 80 metres. The wheel had 36 wooden carts on the wheel, with each cart able to hold approximately 60 people

The Ferris Wheel was introduced to the world at the 1893 World's Fair in Chicago. The illustration shown below is a copy of a photograph of the original wheel.

Time (t) minutes	Height (h) metres
0	4
2.25	42
4.5	80
6.75	42
9	4

a) If the wheel rotated every nine minutes, use the data in the table to sketch a sinusoidal graph which represents the height of a car in metres, as a function of time in minutes. Assume that the car is at its lowest point at $t = 0$, and draw one complete cycle.



b) Determine the equation of the graph in the form $h(t) = a\cos b(t-c) + d$.

amplitude $= \frac{80-4}{2} = 38$ m $\quad a = 38$

period $= 9$ min $\quad b = \frac{2\pi}{9}$

h.p.s. $= 4.5$ min right $\quad c = 4.5$

vert. disp $= \frac{80+4}{2} = 42$ m $\quad d = 42$

$$h(t) = 38\cos\tfrac{2\pi}{9}(t-4.5) + 42$$

c) How high, to the nearest metre, is the cart 5 minutes after the wheel starts rotating?

$h(5) = 38\cos\frac{2\pi}{9}(5-4.5) + 42 = 77.708...$ $\quad$ 78 metres

d) How many seconds after the wheel starts rotating does the cart first reach 10 metres from the ground? Answer to the nearest second.

$10 = 38\cos\frac{2\pi}{9}(t-4.5) + 42$

graph $y_1 = 38\cos\frac{2\pi}{9}(x-4.5) + 42$

graph $y_2 = 10$

first intersection point $x = 0.8159..$

$0.8159.. \times 60 = 48.955...$

$= 49$ sec.

Complete Assignment Questions #1 - #5