Trigonometry - Equations and Identities Lesson #4:
Trigonometric Identities - Part One

Equations and Identities

In mathematics it is important to understand the difference between an equation and an identity.

$2x^2 + 3 = 11$ is an **equation**. It is only true for <u>certain values</u> of the variable x.
The solutions to this equation are -2 and 2 which can be verified by substituting these values into the equation.

$(x + 1)^2 = x^2 + 2x + 1$ is an **identity**. It is true for <u>all values</u> of the variable x.

Reviewing Identities

Recall the basic trigonometric identities:

Basic Identities

$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$ $\quad$ where $x^2 + y^2 = r^2$

We have also met the reciprocal trigonometric identities :

Reciprocal Identities

$\csc x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$ $\quad \cot x = \frac{1}{\tan x}$

We can use the Basic and Reciprocal trigonometric identities to prove the Quotient and Pythagorean identities.

Before doing this we will verify some identities using a particular case.

Verifying Identities for a Particular Case

When verifying an identity we must treat the left side (LS) and the right side (RS) **separately** and work until **both sides** represent the same **value**.

This technique **does not prove** that an identity is true for **all** values of the variable - only for the value of the variable being verified.

Verify the following identities for the value given.

a) $\tan x = \dfrac{\sin x}{\cos x}$ for $x = 60°$

L.S.	R.S.

b) $\tan^2 x + 1 = \sec^2 x$ for $x = \dfrac{\pi}{6}$

L.S.	R.S.

Proving the Quotient Identities and the Pythagorean Identities

We can use identities to derive other identities. When proving an identity we must:

- treat the left side (LS) and the right side (RS) **separately**
- work until **both sides** represent the **same expression**.

Remember:

- Do not make the mistake of assuming the answer by writing the LS = RS at the start of a proof and do not move terms from one side to the other.

Use the basic identities to prove the identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, where $\cos\theta \neq 0$. Explain why there is the restriction $\cos\theta \neq 0$.

L.S.	R.S.

Use the basic identities to prove the identity $1 + \tan^2 A = \sec^2 A$.

In the same way the basic identities can be used to prove the following:

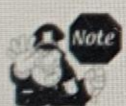
Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

and

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \qquad 1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$



- These identities can be written in several ways and this should be remembered in trying to prove more difficult identities in the next lesson. For example

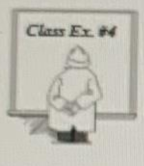
$$\sin^2 x = 1 - \cos^2 x \qquad \cos^2 x = 1 - \sin^2 x$$

$$\tan^2 x = \sec^2 x - 1 \qquad \cot^2 x = \csc^2 x - 1 \qquad \text{etc.}$$

- We use the basic trigonometric identities in terms of x, y and r to prove **only** the Quotient and Pythagorean Identities.
- You will be asked to verify the remaining Quotient and Pythagorean Identities in the Assignment.
- Before considering more complex identities in the next lesson we need to review some skills in simplification and factoring which will help in the proofs.

Complete Assignment Questions #1 - #5

Using Identities to Simplify Trigonometric Expressions



Express each as a single trigonometric ratio. Use a graphing calculator to verify.

a) $\dfrac{\sin^2 x}{\cos^2 x} + 1$

b) $\sin x + \cot x \cos x$

Express $\dfrac{2\tan A}{1 + \tan^2 A}$ in terms of $\sin A$ and $\cos A$ and write in simplest form.

Factor the following trigonometric expressions.

a) $3\cos^4\theta - 3\sin^4\theta$

b) $\sin^2\theta + \sin^2\theta\cot^2\theta$

Complete Assignment Questions #6 - #17

Assignment

1. Verify the following identities for the given value of the variable.

a) $\cot x = \dfrac{\cos x}{\sin x}$ for $x = 60°$

b) $\sin^2 x + \cos^2 x = 1$ for $x = \dfrac{\pi}{4}$

2. Verify the identity $1 + \cot^2 x = \csc^2 x$ for the given values:

   a) $x = \frac{\pi}{6}$

   b) $120°$

3. Explain why verifying that the two sides of a trigonometric identity are equal for given values (as in #1 and #2 above) is insufficient to conclude that the identity is valid.

4. Use the basic identities to prove the identities in questions #1 and #2.

5. Use the quotient identities or the Pythagorean identities to state whether the following are true or false.

   a) $\cos^2 x = 1 + \sin^2 x$

   b) $(\sin x)(\csc x) = 1$

   c) $\sin x = \pm\sqrt{1 - \cos^2 x}$

   d) $(\tan x)(\cot x) = 1$

   e) $\tan^2 x - \sec^2 x = 1$

6. Write each expression as a single trigonometric ratio or as the number 1.

a) $\sin^2 x - 1$

b) $\dfrac{\cos t}{\sin t}$

c) $\dfrac{1}{\sec \theta}$

d) $(\sec t)(\sin^2 t)(\csc t)$

e) $\csc^2 x - \cot^2 x$

f) $\sin \theta + (\cot \theta)(\cos \theta)$

7. Factor to write each in a simpler form.

a) $\sec x \sin^2 x - \sec x$

b) $\sin^4 \theta - \cos^4 \theta$

8. Very often in proving identities it is simpler to try to express each side in terms of only $\sin x$, $\cos x$, or both. Express each of the following in terms of only $\sin x$, $\cos x$, or both.

a) $\tan^2 x$

b) $\dfrac{\tan x}{\sin x}$

c) $\dfrac{\tan x}{\csc x}$

d) $\dfrac{1}{1 + \cot^2 x}$

e) $\csc x - \sin x$

f) $1 - \csc^2 x$

g) $\dfrac{1 + \cot^2 x}{\sec^2 x}$

h) $\dfrac{\cos^2 x - 1}{\tan x}$

i) $\dfrac{1 + \cot^2 x}{\cot^2 x}$

Multiple Choice **In questions #9 - #14 assume the appropriate restrictions.**

9. $\dfrac{\cos x}{1 - \sin^2 x}$ is equal to

A. $\sec x$

B. $\csc x$

C. $\sin x$

D. $\tan x$

10. $\dfrac{\tan^2 x + 1}{\sec x}$ is equal to

A. $\sec x$

B. $\csc x$

C. $\sin x$

D. $\tan x$

11. $\dfrac{\csc x}{\cot x}$ is equal to

A. $\cos x$

B. $\sin x$

C. $\sec x$

D. $\tan x$

12. Which is **not** an identity?

A. $\cos^2 x + \sin^2 x = 1$

B. $\sin x + \cos x = 1$

C. $\sec^2 x - \tan^2 x = 1$

D. $\tan x \cot x = 1$

13. $\sec x - \cos x$ is equal to

A. $\dfrac{1 - \cos x}{\cos x}$

B. $\dfrac{1 - 2\cos x}{\cos x}$

C. $\sin^2 x$

D. $\sin x \tan x$

14. The expression $\dfrac{\tan A \cos^2 A}{\sec A}$, expressed in terms of $\sin A$, is

A. $\dfrac{\sin A}{1 - \sin^2 A}$

B. $\dfrac{1 - \sin^2 A}{\sin A}$

C. $\sin^2 A$

D. $\sin A - \sin^3 A$

15. If $\tan x \neq 0$, $\cos x \neq 0$, $\cot x \neq 0$, then $\dfrac{1}{\tan x \cos x \cot x}$ is equal to

A. $\dfrac{1}{\sin x}$

B. $\sin x$

C. $\dfrac{1}{\cos x}$

D. $\cos x$

16. If $\sin x \neq 0$, $\cos x \neq 0$, then $\dfrac{\tan x \cos x}{3 \sec x \cot x}$ is equal to

A. $\dfrac{1}{3}$

B. 3

C. $\dfrac{1}{3}\sin^2 x$

D. $\dfrac{1}{3}\csc^2 x$

Numerical Response

17. When verifying the identity $\cot^2 x + 1 = \csc^2 x$ for $x = \dfrac{\pi}{7}$, the value on each side of the identity, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right.)

Answer Key

3. The left side and right side may be equal for some values, but may be unequal for other values. An identity is only valid if the left side and the right side are equal for all values for which the identity is defined.

5. b), c) and d) are true. 6. a) $-\cos^2 x$ b) $\cot t$ c) $\cos\theta$ d) $\tan t$ e) 1 f) $\csc\theta$

7. a) $-\cos x$ b) $(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)$

8. a) $\dfrac{\sin^2 x}{\cos^2 x}$ b) $\dfrac{1}{\cos x}$ c) $\dfrac{\sin^2 x}{\cos x}$ d) $\sin^2 x$ e) $\dfrac{\cos^2 x}{\sin x}$

f) $-\dfrac{\cos^2 x}{\sin^2 x}$ g) $\dfrac{\cos^2 x}{\sin^2 x}$ h) $-\sin x \cos x$ i) $\dfrac{1}{\cos^2 x}$

9. A 10. A 11. C 12. B 13. D 14. D

15. C 16. C 17.

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