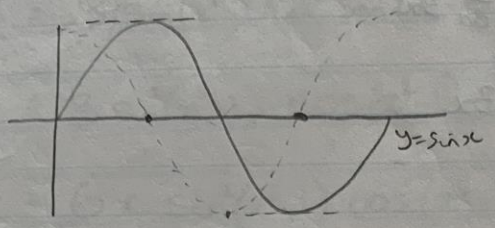
Derivatives of Sine and Cosine Functions



$$\boxed{\frac{d}{dx}\sin x = \cos x}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

addition Rule

$$= \lim_{h\to 0} \frac{\sin x\cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$$

$\lim_{h\to 0} \sin h = 0$

$\lim_{h\to 0} \cos h = 1$

$\lim_{h\to 0} \frac{\sin h}{h} = 1$

$$= \sin x \lim_{h\to 0} \frac{\cos h - 1}{h} + \cos x \lim_{h\to 0} \frac{\sin h}{h}$$

$$= \sin x(0) + \cos x(1)$$

$$= \cos x$$

$$\boxed{\frac{d}{dx}\cos x = -\sin x}$$

$f(x) = \cos x$

$= \sin\left(\frac{\pi}{2} - x\right)$

$f'(x) = \cos\left(\frac{\pi}{2} - x\right)\frac{d}{dx}\left(\frac{\pi}{2} - x\right)$

$= \sin x(-1)$

$= -\sin x$

P.T.O

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Examples Always be aware of the chain rule when differentiating trig functions. function of a function

① Differentiate $y = \sin 3x$

$$\frac{dy}{dx} = \cos 3x \frac{d}{dx} 3x$$

$$= 3\cos 3x$$

---

② $y = \sin(x+2)$

$$\frac{dy}{dx} = \cos(x+2)\frac{d}{dx}(x+2)$$

$$= \cos(x+2)$$

---

③ $y = \sin(kx+d)$

$$\frac{dy}{dx} = \cos(kx+d)\frac{d}{dx}(kx+d)$$

$$= k\cos(kx+d)$$

---

④ $y = \sin(x^3)$

$$\frac{dy}{dx} = \cos x^3 \frac{d}{dx} x^3$$

$$= 3x^2 \cos x^3$$

---

⑤ $y = \sin^3 x$

$y = (\sin x)^3$

let $u = \sin x$

$y = u^3$

$$\frac{dy}{dx} = 3(\sin x)^2 \frac{d}{dx} \sin x$$

$$= 3\sin^2 x \cos x$$

⑥ $y = \sin^3(x^2-1)$

$y = [\sin(x^2-1)]^3$

let $u = \sin(x^2-1)$

$\frac{dy}{dx} = 3[\sin(x^2-1)]^2 \frac{d}{dx} \sin(x^2-1)$

$= 3\sin^2(x^2-1)\cos(x^2-1)\frac{d}{dx}(x^2-1)$

$= 6x\sin^2(x^2-1)\cos(x^2-1)$

⑦ $y = x^2\cos x$

use product rule

$(fg)' = fg' + gf'$

$f = x^2 \quad g = \cos x$

$f' = 2x \quad g' = -\sin x$

$\frac{dy}{dx} = x^2(-\sin x) + \cos x(2x)$

$= -x^2\sin x + 2x\cos x$

⑧ If $\sin x + \sin y = 1$ find the derivative with respect to x

$\frac{dy}{dx}\sin x + \frac{d}{dy}\sin y \frac{dy}{dx} = \frac{dy}{dx}1$

$\cos x + \cos y \frac{dy}{dx} = 0$

$\cos y \frac{dy}{dx} = -\cos x$

$\frac{dy}{dx} = \frac{-\cos x}{\cos y}$

P. 313 # 1 alt, 2 alt