Derivatives of Other Trigonometric Functions

$$\boxed{\frac{d}{dx}\tan x = \sec^2 x}$$

Proof $y = \tan x = \dfrac{\sin x}{\cos x}$ Quotient Rule $\dfrac{gf' - fg'}{g^2}$

$f = \sin \quad g = \cos$

$f' = \cos \quad g' = -\sin$

$$\frac{dy}{dx} = \frac{\cos x \cos x + \sin x \sin x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\boxed{\frac{d}{dx}\csc x = -\csc x \cot x}$$

Proof $y = \csc x = \dfrac{1}{\sin x} = (\sin x)^{-1}$

$$\frac{dy}{dx} = -(\sin x)^{-2} \frac{d}{dx} \sin x$$

$$= -(\sin x)^{-2} \cos x$$

$$= -\frac{1}{\sin^2 x} \cos x$$

$$= -\frac{1}{\sin x} \frac{\cos x}{\sin x}$$

$$= -\csc x \cot x$$

Other Identities on P. 316

eg1 Differentiate $y = 2\csc^3(3x^2)$

$$y = 2(\csc(3x^2))^3$$

$$\frac{dy}{dx} = 6[\csc(3x^2)]^2 \frac{d}{dx}\csc(3x^2)$$

$$= 6\csc^2(3x^2)[-\csc(3x^2)]\cot(3x^2)\frac{d}{dx}3x^2$$

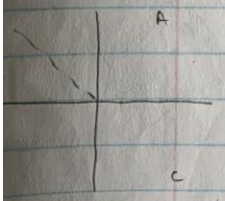
$$= -36x\csc^3(3x^2)\cot(3x^2)$$

eg2 Find the equation of the tangent line $y = \sec x + \csc x$ when $x = \frac{3\pi}{4}$

$$\frac{dy}{dx} = \sec x \tan x - \csc x \cot x$$

$$y - y_1 = m(x - x_1)$$

When $x = \frac{3\pi}{4}$ $\quad y = \sec\frac{3\pi}{4} + \csc\frac{3\pi}{4}$



$$= \frac{1}{\cos\frac{3\pi}{4}} + \frac{1}{\sin\frac{3\pi}{4}}$$

$$= -\sqrt{2} + \sqrt{2}$$

$$= 0$$

$$m = \sec\frac{3\pi}{4}\tan\frac{3\pi}{4} - \csc\frac{3\pi}{4}\cot\frac{3\pi}{4}$$

$$= \frac{1}{\cos\frac{3\pi}{4}}\tan\frac{3\pi}{4} - \frac{1}{\sin\frac{3\pi}{4}}\frac{1}{\tan\frac{3\pi}{4}}$$

$$= (-\sqrt{2})(-1) - (\sqrt{2})(-1)$$

$$= 2\sqrt{2}$$

$$y - 0 = 2\sqrt{2}\left(x - \frac{3\pi}{4}\right)$$

$$y = 2\sqrt{2}x - \frac{6\sqrt{2}\pi}{4}$$

$$y = 2\sqrt{2}x - \frac{3\sqrt{2}\pi}{2}$$

$$2y = 4\sqrt{2}x - 3\sqrt{2}\pi$$

$$4\sqrt{2}x - 2y - 3\sqrt{2}\pi = 0$$

Complete Questions:

Page 313 # 1bdfhjlnprt, 10, 11a

Page 319 # 1acegikmo, 3ac, 13a