Derivatives of Inverse Trigonometric functions

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

eg1 $y = \cos^{-1}(x^2)$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-(x^2)^2}}\frac{d}{dx}x^2$$

$$= \frac{-2x}{\sqrt{1-x^4}}$$

eg2 $y = (1+x^2)\tan^{-1}x$

$fg' + gf'$

$f = (1+x^2)$ $\quad g = \tan^{-1}x$

$f' = 2x$ $\quad g' = \frac{1}{1+x^2}$

$$\frac{dy}{dx} = \frac{1+x^2}{1+x^2} + 2x\tan^{-1}x$$

$$= 1 + 2x\tan^{-1}x$$

eg 3 $y = \frac{\sin^{-1}x}{\cos^{-1}x}$

$\frac{gf' - fg'}{g^2}$

$f = \sin^{-1}x \qquad g = \cos^{-1}x$

$f' = \frac{1}{\sqrt{1-x^2}} \qquad g' = -\frac{1}{\sqrt{1-x^2}}$

$$\frac{dy}{dx} = \frac{\cos^{-1}x\left(\frac{1}{\sqrt{1-x^2}}\right) - \sin^{-1}x\left(-\frac{1}{\sqrt{1-x^2}}\right)}{(\cos^{-1}x)^2}$$

$$= \frac{\frac{\cos^{-1}x}{\sqrt{1-x^2}} - \frac{-\sin^{-1}x}{\sqrt{1-x^2}}}{(\cos^{-1}x)^2}$$

$$= \frac{\cos^{-1}x + \sin^{-1}x}{(\cos^{-1}x)^2\sqrt{1-x^2}}$$

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