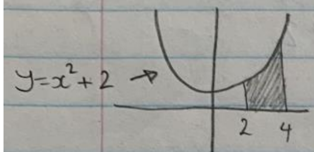
Area Under a Curve

example ①

Find the area under $y = x^2 + 2$ from $x = 2$ to $x = 4$



$$y = x^2 + 2$$

$$\int_2^4 x^2 + 2 = \left[\frac{1}{3}x^3 + 2x\right]_2^4$$

$$\left(\tfrac{1}{3}x^3 + 2x + c\right) - \left(\tfrac{1}{3}x^3 + 2x + c\right) = \left(\tfrac{1}{3}(4)^3 + 2(4)\right) - \left(\tfrac{1}{3}(2)^3 + 2(2)\right)$$

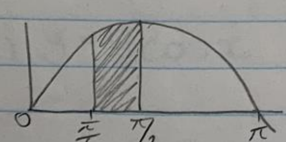
$$= 64/3 + 8 - 8/3 - 4$$

$$= 56/3 + 4$$

$$= 68/3$$

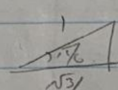
example ②

Find the area between $y = \sin x$ and the x-axis from $x = \pi/6$ to $x = \pi/2$



$$\int_{\pi/6}^{\pi/2} \sin x = \left[-\cos x\right]_{\pi/6}^{\pi/2}$$

$$= (-\cos \pi/2) - (-\cos \pi/6)$$



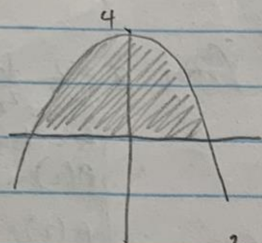
$$= 0 + \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2}$$

See the last page for why the 'C' has magically disappeared from consideration.

example ③

Find the area of the region below $y = 4 - x^2$ and above the x-axis.



Solve for x-intercepts: $0 = 4 - x^2$

$x^2 = 4$

$x = \pm 2$

$$\int_{-2}^{2} 4 - x^2 = \left[4x - \tfrac{1}{3}x^3\right]_{-2}^{2}$$

$$= \left[4(2) - \tfrac{1}{3}(2)^3\right] - \left[4(-2) - \tfrac{1}{3}(-2)^3\right]$$

$$= \left(8 - 8/3\right) - \left(-8 + 8/3\right)$$

$$= 16 - 16/3$$

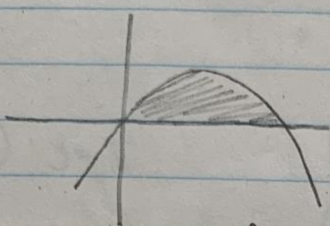
$$= 48/3 - 16/3$$

$$= 32/3$$

Same topic, extra examples:

Area Under a Curve

eg1 Find the area between the curve of $y = 2x - x^2$ and the x-axis.



$$0 = 2x - x^2$$
$$0 = x(2-x)$$
$$x = 0 \quad x = 2$$

$$\int_0^2 2x - x^2 = \left[x^2 - \frac{1}{3}x^3 \right]_0^2$$

$$\text{Area} = \left[2^2 - \frac{1}{3}(2)^3\right] - \left[0^2 - \frac{1}{3}(0)^3\right]$$

$$= 4 - 8/3 \quad - \quad 0$$

$$= 4/3$$

eg2 Find the area under $y = \frac{1}{x^2}$ from $x = 2$ to $x = 5$

$$\int_2^5 \frac{1}{x^2} = \int_2^5 x^{-2} = \left[-x^{-1} \right]_2^5 = \left[-\frac{1}{x} \right]_2^5$$

$$= \left[-\frac{1}{5}\right] - \left[-\frac{1}{2}\right]$$

$$= -\frac{1}{5} + \frac{1}{2}$$

$$= \frac{3}{10}$$

Why is 'C' not important?

For an indefinite integral:

$$\int 2x - x^2 = x^2 - \frac{1}{3}x^3 + C$$

We now have defined values and subtracting them from each other to find an area:

$$\int_0^2 2x - x^2 = \left[x^2 - \frac{1}{3}x^3 + C\right]_0^2$$

Substitute 2 and 0 into our integral

$$\left(2^2 - \frac{1}{3}(2)^3 + C\right) - \left(0^2 - \frac{1}{3}(0)^3 + C\right)$$

$$= 4 - \frac{8}{3} + C \quad - (0 - 0 + C)$$

$$= \frac{4}{3} \quad +C \quad - \quad C$$

the 'C' value will always cancel out.