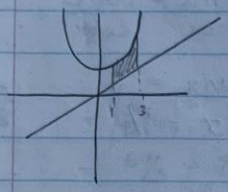
Area Between Curves

Example ① Find the area between $y = x^2 + 1$ and $y = x$ from $x = 1$ to $x = 3$



$$A = x^2 + 1 - x = x^2 - x + 1$$

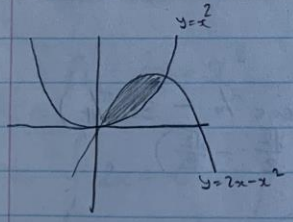
$$\int_1^3 x^2 - x + 1 = \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 + x \right]_1^3$$

$$= \left(9 - \frac{9}{2} + 3\right) - \left(\frac{1}{3} - \frac{1}{2} + 1\right)$$

$$= \frac{20}{3}$$

Example ②

Find the area of the region bounded by the parabolas $y = x^2$ and $y = 2x - x^2$



PoI $\quad x^2 = 2x - x^2$

$$2x^2 - 2x = 0$$

$$2x(x - 1) = 0$$

$$x = 0 \quad x = 1$$

order matters! line on top goes first

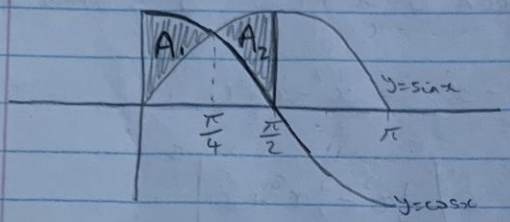
$$\int_0^1 2x - x^2 - x^2 = \int_0^1 2x - 2x^2 = \left[x^2 - \frac{2}{3}x^3 \right]_0^1$$

$$= (1 - 2/3) - (0)$$

$$= 1/3$$

Example ③

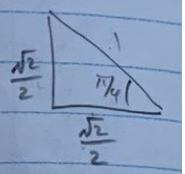
Find the area of the region between the curves $y = \sin x$ and $y = \cos x$ from 0 to $\pi/2$.



PoI $\quad \dfrac{\sin x}{\cos x} = \dfrac{\cos x}{\cos x}$

$\tan x = 1$

$x = \pi/4$



$$A_1 \int_0^{\pi/4} \cos x - \sin x = \left[\sin x + \cos x\right]_0^{\pi/4}$$

$$= (\sin \pi/4 + \cos \pi/4) - (\sin 0 + \cos 0)$$

$$= (\sqrt{2}/2 + \sqrt{2}/2) - (0 + 1)$$

$$= \sqrt{2} - 1$$

$$A_2 \int_{\pi/4}^{\pi/2} \sin x - \cos x = \left[-\cos x - \sin x\right]_{\pi/4}^{\pi/2}$$

$$= (-\cos \pi/2 - \sin \pi/2) - (-\cos \pi/4 - \sin \pi/4)$$

$$= (0 - 1) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)$$

$$= -1 + \sqrt{2}$$

$$= \sqrt{2} - 1$$

Total Area $= \sqrt{2} - 1 + \sqrt{2} - 1$

$$= 2\sqrt{2} - 2$$