Calculus 12 – Trigonometric Identities

All identities listed on P.281/P.282

Proving Trigonometric Identities

example 1

Prove $1 + \cos x = \dfrac{\sin^2 x}{1 - \cos x}$

$\sin^2 x + \cos^2 x = 1$

move from complex to simple

$$= \frac{1 - \cos^2 x}{1 - \cos x}$$

$$= \frac{\cancel{(1 - \cos x)}(1 + \cos x)}{\cancel{1 - \cos x}}$$

$$= 1 + \cos x$$

example 2

Prove $\cos(x+y)\cos(x-y) = \cos^2 x + \cos^2 y - 1$

$$= (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)$$

Remove sin as not on left

$$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$$

$$= \cos^2 x \cos^2 y - (1 - \cos^2 x)(1 - \cos^2 y)$$

$$= \cos^2 x \cos^2 y - (1 - \cos^2 y - \cos^2 x + \cos^2 x \cos^2 y)$$

$$= \cancel{\cos^2 x \cos^2 y} - 1 + \cos^2 y + \cos^2 x - \cancel{\cos^2 x \cos^2 y}$$

$$= \cos^2 x + \cos^2 y - 1$$

Hilroy

aim to get as basic sin and cos functions

Example 3

Prove $\frac{\sin 2x}{1-\cos 2x} = 2\csc 2x - \tan x$

$\frac{2\sin x\cos x}{1-(1-2\sin^2 x)} = \frac{2}{\sin 2x} - \tan x$

$\frac{\cancel{2}\,\cancel{\sin x}\cos x}{\cancel{2}\sin^{\cancel{2}} x} = \frac{2}{2\sin x\cos x} - \frac{\sin x}{\cos x}$

$\frac{\cos x}{\sin x} = \frac{1}{\sin x\cos x} - \frac{\sin^2 x}{\sin x\cos x}$

$\frac{1-\sin^2 x}{\sin x\cos x}$

$\frac{\cos^{\cancel{2}} x}{\sin x\cancel{\cos x}}$

$\frac{\cos x}{\sin x}$

$\therefore \frac{\sin 2x}{1-\cos 2x} = 2\csc 2x - \tan x$

P. 284 # 1-5, 10-14, 26-29

Extra Examples

① $\dfrac{\sin x + \sin^2 x}{\cos x + \sin x \cos x} = \tan x$

$\dfrac{\sin x\,\cancel{(1 + \sin x)}}{\cos x\,\cancel{(1 + \sin x)}} = \tan x$

$\dfrac{\sin x}{\cos x} = \tan x$

$\tan x = \tan x$

② $\dfrac{1 - \sin^2 x}{\cos x} = \dfrac{\sin 2x}{2 \sin x}$

$\dfrac{\cos^2 x}{\cos x} = \dfrac{\cancel{2 \sin x} \cos x}{\cancel{2 \sin x}}$

$\cos x = \cos x$

③ $\dfrac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$

$\dfrac{\csc^2 x}{\csc^2 x} - \dfrac{1}{\csc^2 x} = \cos^2 x$

$1 - \dfrac{1}{\csc^2 x} = \cos^2 x$

$1 - \sin^2 x = \cos^2 x$ $\qquad \cos^2 x + \sin^2 x = 1$

$\cos^2 x = \cos^2 x$