Antiderivatives (Integration)

This is the reverse process of differentiation.

$$\frac{1}{n+1}x^{n+1}$$

Examples

Find the antiderivative of:

C is a constant that may or may not exist. $\frac{dy}{dx}C = 0$. Reversing the process we have to assume a constant may exist.

i) $f(x) = 2x^2 - x + 7$

$$F(x) = 2\left(\frac{1}{3}x^3\right) - \frac{1}{2}x^2 + 7(x) + C$$

$$= \frac{2}{3}x^3 - \frac{1}{2}x^2 + 7x + C$$

Proof P. 376 $\frac{d}{dx}\ln x = \frac{1}{x}$

ii) $f(x) = \frac{2}{x^2} - \frac{5}{x} + x = 2x^{-2} - 5\left(\frac{1}{x}\right) + x$

table on P. 405

$$F(x) = 2\left(\frac{1}{-2+1}x^{-2+1}\right) - 5\ln|x| + \frac{1}{2}x^2 + C$$

$$= 2(-1x^{-1}) - 5\ln|x| + \frac{1}{2}x^2 + C$$

$$= -\frac{2}{x} - 5\ln|x| + \frac{1}{2}x^2 + C$$

Trig/Exponential Antiderivatives

Find the antiderivative of :

i) $f(x) = \cos x - \sin x$

$F(x) = \sin x - (-\cos x) + C$

$= \sin x + \cos x + C$

antiderivative

$e^{kx} \rightarrow \frac{1}{k} e^{kx}$

ii) $f(x) = -3e^{-x} + 6e^{2x}$

$F(x) = -3\left(\frac{1}{-1} e^{-x}\right) + 6\left(\frac{1}{2} e^{2x}\right) + C$

$= 3e^{-x} + 3e^{2x} + C$

iii) $f(x) = \sin x + \frac{1}{x^3} = \sin x + x^{-3}$

$F(x) = -\cos x + \frac{1}{-3+1} x^{-3+1} + C$

$= -\cos x - \frac{1}{2} x^{-2} + C$

$= -\cos x - \frac{1}{2x^2} + C$

P. 405 for table to help

iv) $f(x) = \sin 3x + 3\cos x$

$F(x) = -\frac{1}{3}\cos 3x + 3\sin x + C$

$\frac{dy}{dx} - \frac{1}{3}\cos 3x$

$= \frac{1}{3}\sin 3x \frac{d}{dx} 3x$

$= \sin 3x$

Page 408 Questions 1 - 5