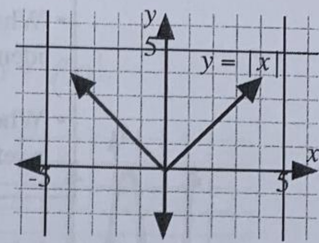
Absolute Value Functions and Reciprocal Functions Lesson #4:
Absolute Value Transformations

Absolute Value Transformations

Recall the definition of absolute value.



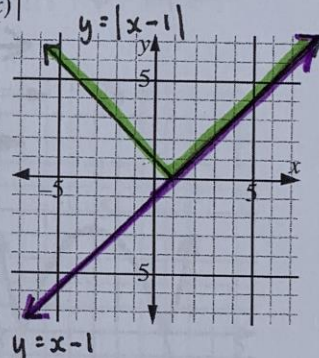
$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

An absolute value transformation transforms the graph of $y = f(x)$ to the graph of $y = |f(x)|$.

Investigating the Graphs of $y = f(x)$ and $y = |f(x)|$

1. A function $f(x)$ has equation $y = x - 1$.

x	* $y = f(x)$	$y = \lvert f(x) \rvert$
-4	-5	5
-3	-4	4
-2	-3	3
-1	-2	2
0	-1	1
1	0	0
2	1	1
3	2	2
4	3	3



a) Write the equation for $y = |f(x)|$.

$y = |x-1|$

b) Complete the table of values for $y = f(x)$ and $y = |f(x)|$.

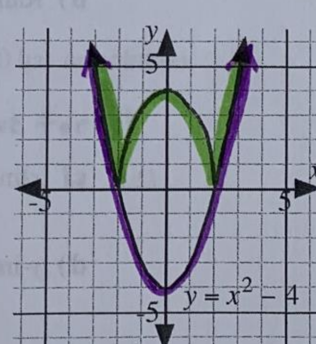
c) Sketch the graphs of $y = f(x)$ and $y = |f(x)|$ on the same grid.

d) Complete the following statements based on the observations in c).

i) When $f(x) \geq 0$, the graph of $y = |f(x)|$ is identical to the graph of $y = f(x)$.

ii) When $f(x) < 0$, the graph of $y = |f(x)|$ is a reflection in the x-axis of the graph of $y = f(x)$.

2. The graph of the function $f(x)$ with equation $y = x^2 - 4$ is * shown.



a) Write the equation for $y = |f(x)|$. $y = |x^2-4|$ *

b) Use a graphing calculator to sketch $y = |f(x)|$.

c) Do the observations from #1d) also apply in this example?

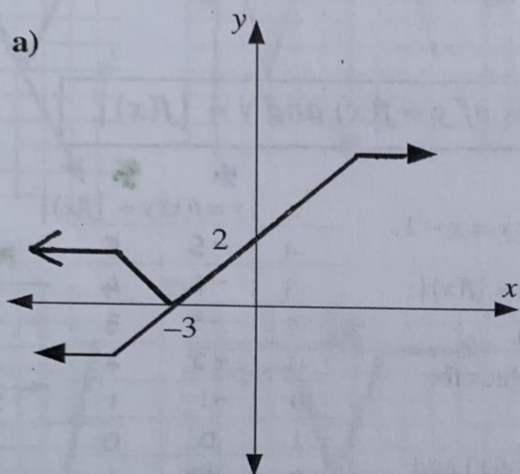
yes

In general, given the function $y = f(x)$, the graph of $y = |f(x)|$ has the following characteristics:

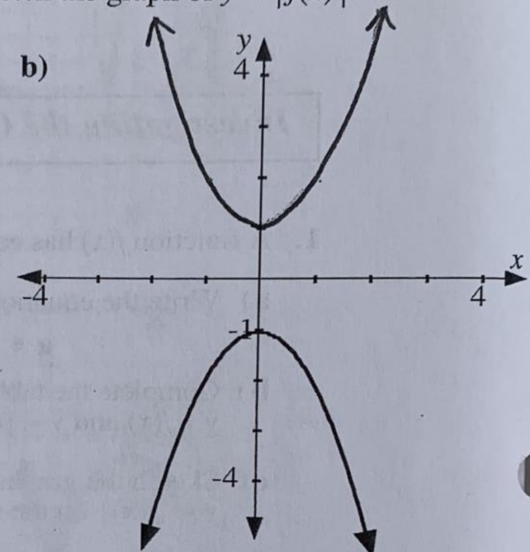
- When $f(x) \geq 0$, (i.e. the graph of $y = f(x)$ is above the x-axis), the graph of $y = |f(x)|$ is identical to the graph of $y = f(x)$.
- When $f(x) < 0$, (i.e. the graph of $y = f(x)$ is below the x-axis), the graph of $y = |f(x)|$ is a reflection of the graph of $y = f(x)$ in the x-axis.

3. In each case, the graph of $y = f(x)$ is shown. Sketch the graph of $y = |f(x)|$.

a)



b)



4. Consider all the graphs of $y = f(x)$ and $y = |f(x)|$ from parts 1 to 3 of the investigation. Compare the following aspects of the graphs of $y = f(x)$ and $y = |f(x)|$.

a) Domain   Same

b) Range   the range of $y = |f(x)|$ includes all values of $f(x)$ when $f(x) > 0$, and all values of $-f(x)$ when $f(x) < 0$.

c) x-intercept(s)   Same

d) y-intercepts   If y_{int} of $f(x)$ is positive, the y_{int} of $|f(x)|$ is the same
If y_{int} of $f(x)$ is negative, the y_{int} of $|f(x)|$ is the opposite value (i.e positive)

Complete Assignment Questions #1 - #7