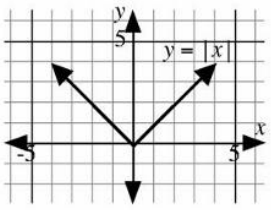
Absolute Value Functions and Reciprocal Functions Lesson #4:
Absolute Value Transformations

Absolute Value Transformations



Recall the definition of absolute value.

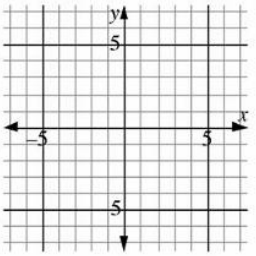
$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

An absolute value transformation transforms the graph of $y = f(x)$ to the graph of $y = |f(x)|$.

Investigating the Graphs of $y = f(x)$ and $y = |f(x)|$

1. A function $f(x)$ has equation $y = x - 1$.

x	$y = f(x)$	$y = \lvert f(x) \rvert$
-4		
-3		
-2		
-1		
0		
1		
2		
3		
4		



a) Write the equation for $y = |f(x)|$.

b) Complete the table of values for $y = f(x)$ and $y = |f(x)|$.

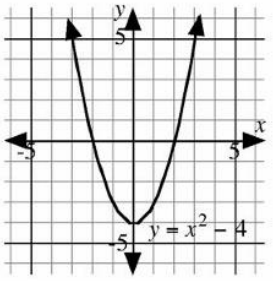
c) Sketch the graphs of $y = f(x)$ and $y = |f(x)|$ on the same grid.

d) Complete the following statements based on the observations in c).

i) When $f(x) \geq 0$, the graph of $y = |f(x)|$ is _____ to the graph of $y = f(x)$.

ii) When $f(x) < 0$, the graph of $y = |f(x)|$ is a _____ of the graph of $y = f(x)$.

2. The graph of the function $f(x)$ with equation $y = x^2 - 4$ is shown.



a) Write the equation for $y = |f(x)|$.

b) Use a graphing calculator to sketch $y = |f(x)|$.

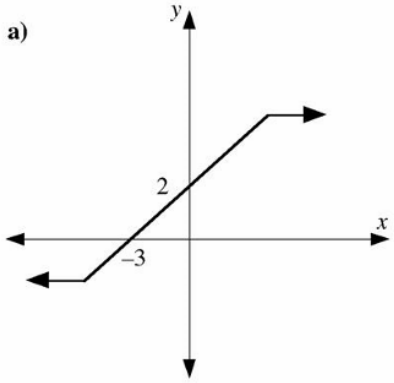
c) Do the observations from #1d) also apply in this example?

In general, given the function $y = f(x)$, the graph of $y = |f(x)|$ has the following characteristics:

- When $f(x) \geq 0$, (i.e. the graph of $y = f(x)$ is above the x-axis), the graph of $y = |f(x)|$ is identical to the graph of $y = f(x)$.
- When $f(x) < 0$, (i.e. the graph of $y = f(x)$ is below the x-axis), the graph of $y = |f(x)|$ is a reflection of the graph of $y = f(x)$ in the x-axis.

3. In each case, the graph of $y = f(x)$ is shown. Sketch the graph of $y = |f(x)|$.

a)



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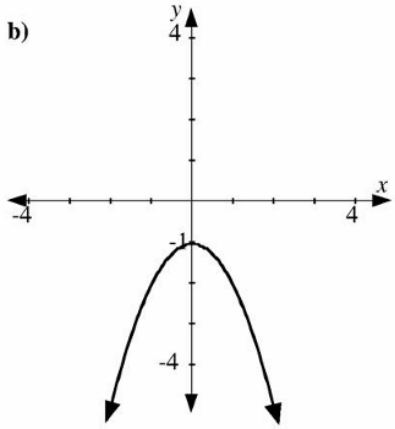


4. Consider all the graphs of $y = f(x)$ and $y = |f(x)|$ from parts 1 to 3 of the investigation. Compare the following aspects of the graphs of $y = f(x)$ and $y = |f(x)|$.

a) Domain

b) Range

c) x-intercept(s)

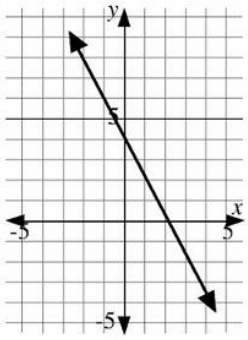
d) y-intercepts

Complete Assignment Questions #1 - #7

Assignment

1. In each case, the graph of $y = f(x)$ is shown. Sketch the graph of $y = |f(x)|$.

a)



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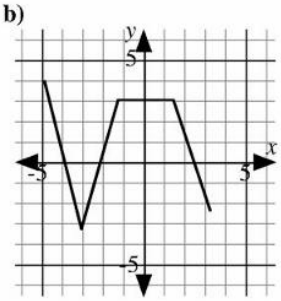

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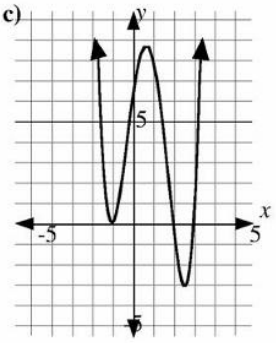

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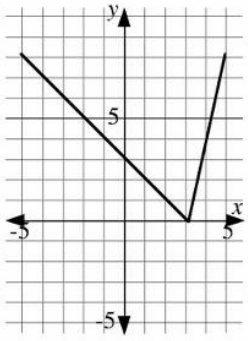
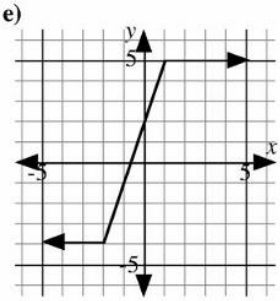


e)



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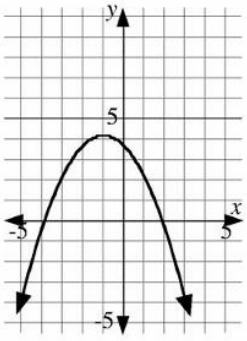
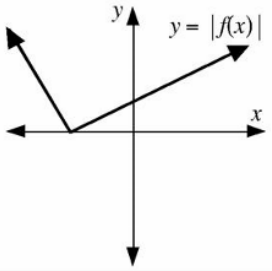


2. Consider a function $y = f(x)$. Explain why the equation $|f(x)| < 0$ has no solution.

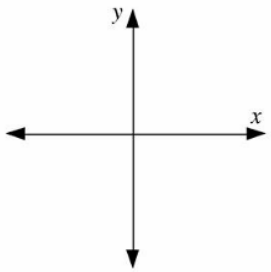
Use the following information to answer the next question.

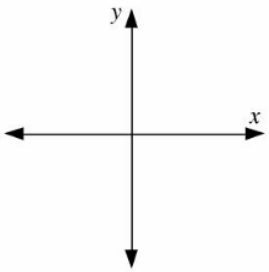
$y = f(x)$ is a continous function with domain $x \in R$.

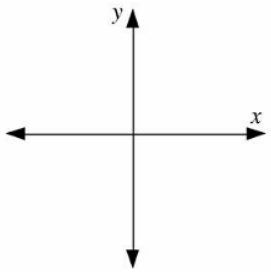
The graph of $y = |f(x)|$ is shown.

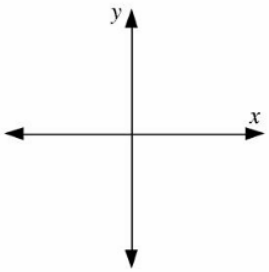


3. Sketch four possible graphs of $y = f(x)$ on the grids below.



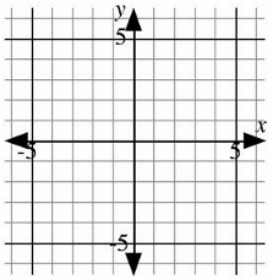






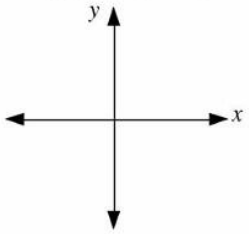
4. The graph of a quadratic function, $g(x)$, has domain $x \in R$, range $y \leq 4$, y-intercept of -5, and x-intercepts 1 and 5.

On the grid provided, sketch the graph of $y = |g(x)|$.

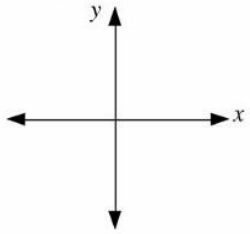


5. Consider a quadratic function $y = f(x)$ with a y-intercept of -2.
Sketch a possible graph of $y = f(x)$ if the range of $y = |f(x)|$ is:

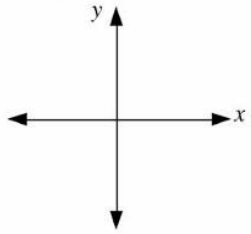
a) $\{y \mid y \geq 0, y \in R\}$



b) $\{y \mid y \geq 2, y \in R\}$



c) $\{y \mid y \geq 1, y \in R\}$



Multiple Choice

6. If the graph of $y = g(x)$ passes through the point $(-3, -6)$, then the graph of $y = |g(x)|$ must pass through the point

A. $(3, -6)$
B. $(3, 6)$
C. $(-3, 6)$
D. $(6, 3)$

7. Which one of the following statements is false?

A. If the y-intercept of the graph of $y = f(x)$ is -8, then the y-intercept of the graph of $y = |f(x)|$ is 8.

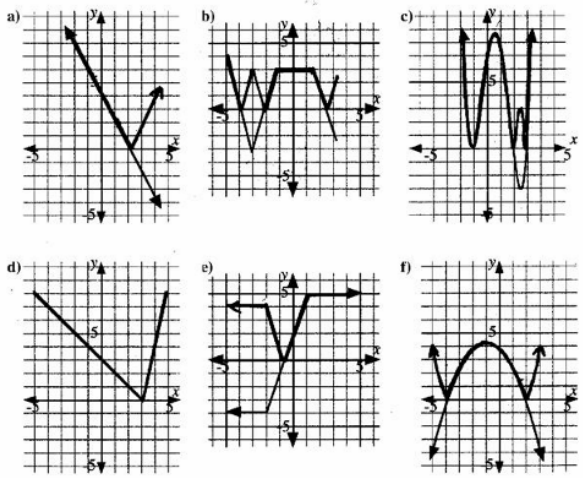
B. If the x-intercepts of the graph of $y = g(x)$ are 2 and 7, then the x-intercepts of the graph of $y = |g(x)|$ are 2 and 7.

C. If the point $(4, 9)$ lies on the graph of $y = |h(x)|$, then the point $(4, -9)$ must lie on the graph of $y = h(x)$.

D. If the graph of a quadratic function $y = P(x)$ crosses the x-axis, then the range of $y = |P(x)|$ must be $\{y \mid y \geq 0, y \in R\}$.

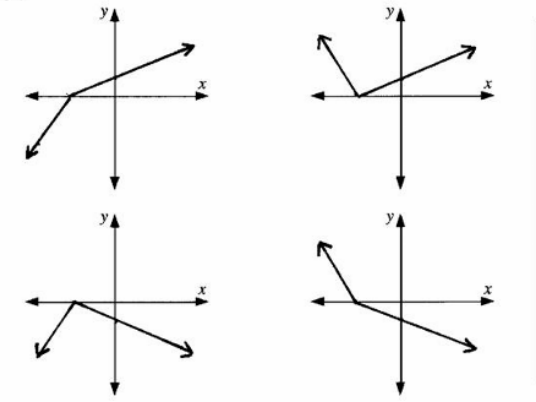
Answer Key

1.



2. The absolute value of a function is non negative for all values of x. It is not possible for $|f(x)|$ to be less than zero.

3.



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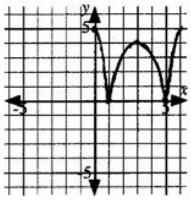
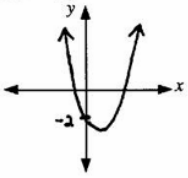
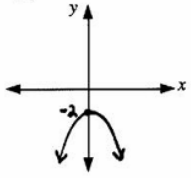


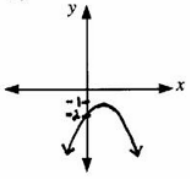
5. a)



b)



c)



6. C **7.** C