

Absolute Value Functions and Reciprocal Functions Lesson #2: Solving Absolute Value Equations - Part One

Absolute Value Equations

$|2x + 3| = 8$, $|3 + x| = 2x + 1$, $|2x - 3| - |x + 4| = 8$, and $|x^2 - 17| = 8$ are all examples of **absolute value equations**.

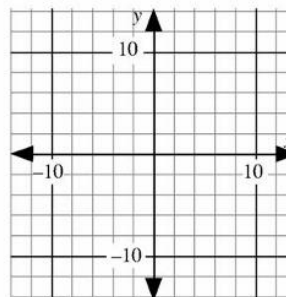
In the next two lessons, we will learn how to solve these equations graphically and algebraically.

Solving Absolute Value Equations Using a Graphing Calculator

Intersection Method

To solve the equation $|2x + 3| = 8$ by the intersection method, use the following procedure.

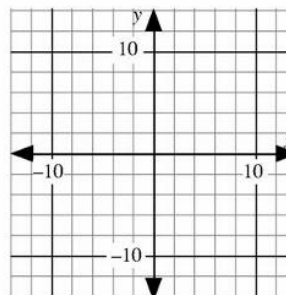
1. Graph $Y_1 = |2x + 3|$.
2. Graph $Y_2 = 8$.
3. Find the x -coordinate(s) of the point(s) of intersection using the `intersect` feature of the calculator.



x-intercept Method

To solve the equation $|2x + 3| = 8$ by the zero method, use the following procedure.

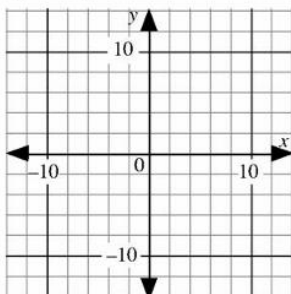
1. Rearrange the original equation with all terms on the left hand side and 0 on the right side to get $|2x + 3| - 8 = 0$.
2. Graph $Y_1 = |2x + 3| - 8$.
3. Use the `zero` feature of the calculator to find the x -intercept(s).



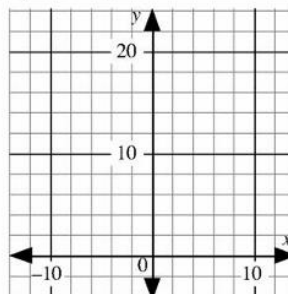


Solve the following absolute value equations by graphing.

a) $|3 + x| = 2x + 1$



b) $|x^2 - 17| = 8$



Complete Assignment Question #1 - #3

Solving Absolute Value Equations Algebraically



There are different ways in which to determine algebraically the solution to absolute value equations.

The method below has the advantage that a virtually identical method can be used to determine the solution to absolute value inequalities in calculus and other higher level math courses.

- Step 1:** Find the value(s) of the variable which will make the expression within the absolute value symbol(s) equal to zero.
- Step 2:** Divide the domain into smaller subdomains using the value(s) found in Step 1.
- Step 3:** Write the absolute value expressions in piecewise form, using the piece that is appropriate for each subdomain.
- Step 4:** Solve the resulting equation in each subdomain.
- Step 5:** Check that the solution to each equation is in the subdomain, and combine all valid solutions.



Class Ex. #2

Alan has started to solve the equation $|2x + 3| = 8$ using the steps on the previous page. Complete Alan's work and check the solution with the graphical solution on page 457.

Alan's solution

subdomain $x < -\frac{3}{2}$	$-\frac{3}{2}$	subdomain $x \geq -\frac{3}{2}$
number line \leftarrow \rightarrow		
Solve $ 2x + 3 = 8$ $-(2x + 3) = 8$		Solve $ 2x + 3 = 8$ $2x + 3 = 8$
Is the solution in the subdomain?		Is the solution in the subdomain?
Final solution: $x =$ _____		



Class Ex. #3

Haley solved the equation $|3 + x| = 2x + 1$ algebraically. Complete Haley's work and compare the solution with the one from Class Ex. #1a).

Haley's solution

subdomain <div style="border: 1px solid black; width: 60px; height: 15px; margin: 0 auto;"></div>	$-\frac{3}{2}$	subdomain <div style="border: 1px solid black; width: 60px; height: 15px; margin: 0 auto;"></div>
number line \leftarrow \rightarrow		
solve $ 3 + x = 2x + 1$		solve $ 3 + x = 2x + 1$
Is the solution in the subdomain?		Is the solution in the subdomain?
Final solution: $x =$ _____		

Complete Assignment Question #4

An Alternative Method for Solving Single Absolute Value Equations

For equations containing a single absolute value expression, such as in Class Ex #2 and #3, and assignment question #4, there is an alternative method which can be used to determine the solution.

When we asked the question “Is the solution in the subdomain?”, the answer was sometimes “yes” and sometimes “no”. An alternative method is to solve the two pieces of the absolute value equation without considering the subdomains. **However, this means that the solution needs to be checked or verified to eliminate incorrect answers.**



Class Ex. #4

Consider the equation $|3 + x| = 2x + 1$.

a) Write $|3 + x|$ as a piecewise expression without using absolute value symbols.

b) Set each of the expressions from a) equal to $2x + 1$, solve for x in each case, and verify.

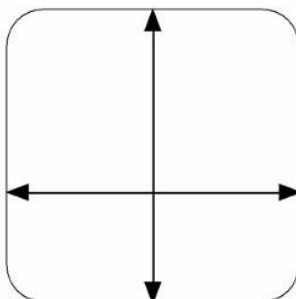
c) Compare this method with the method in Class Ex #3.



Class Ex. #5

Consider the equation $|x^2 - 9| = 7$.

a) Solve the equation by graphing. b) Solve the equation algebraically, using the method in Class Ex #4 b).



Complete Assignment Question #5 - #9

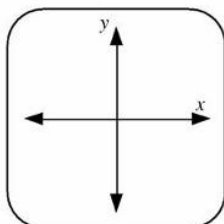
Assignment

1. Describe clearly how to use the method of intersection to solve the equation $|x + 3| = 4$. State the solution.

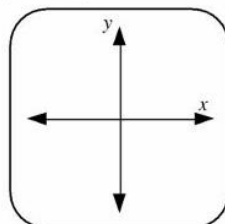
2. Describe clearly how to use the x -intercept method to solve the equation $|x - 2| = x + 1$. State the solution.

3. Solve each of the following equations graphically. Sketch and label each graph.

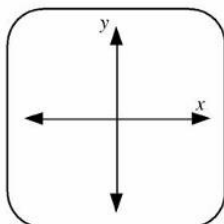
a) $|x + 5| = 0$



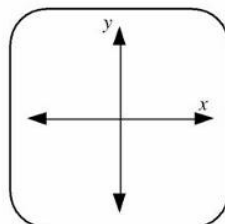
b) $|1 - 4x| = x + 4$



c) $|4 - x| = -2x - 10$



d) $3|x - 8| = 2x + 7$



4. Solve each of the following equations algebraically.

a) $|x + 5| = 10$

b) $|3x - 1| = 4$

c) $|2x + 1| = x$

d) $|1 - 4x| = 6x$

e) $|7x - 2| + 6 = 3x$

f) $|4 - x| = -2x - 10$

5. Solve the following algebraically.

a) $3|x - 8| = 2x + 7$

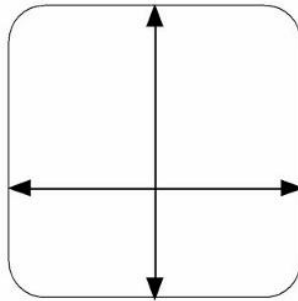
b) $|2x - 8| - 2 = 4x$

c) $|x^2 - 26| = 10$

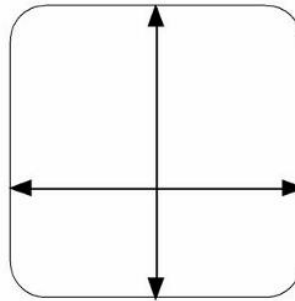
d) $|x^2 + 10x + 15| = 6$

6. Use a graphing calculator to solve, to the nearest tenth, the following absolute value equations.

a) $|x^2 - 2x - 6| = 4$



b) $|12 + 3x - x^2| - 14 = 0$



7. Solve the equation $|x^2 - 2x - 6| = 4$ (from #6a) algebraically, using the quadratic formula.

Multiple Choice 8. The complete solution to $|x| - 4 = 10$ is

- A. 14
- B. 6 and 6
- C. 6 and 14
- D. -14 and 14

**Numerical
Response**

9. Mike solved the equation $|x^2 + 4x - 15| = 6$ algebraically.
 He wrote the solutions as $x = a$, $x = b$, and $x = c \pm \sqrt{d}$.
 The value of $a + b + c + d$ is _____.

(Record your answer in the numerical response box from left to right)

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**Group
Work**

- Some students were trying to solve the equation $|2x + 1| - |x - 2| = 2$.
 Develop and explain an algebraic procedure to solve this equation.
 Determine the solution to the equation.

Answer Key

1. • Graph $Y_1 = |x + 3|$
 • Graph $Y_2 = 4$
 • Find the x -coordinate(s) of the point(s) of intersection using the intersect feature of the calculator.
 • Solution is $x = -7$ or 1 .
2. • Graph $Y_1 = |x - 2| - x - 1$
 • Use the zero feature of the calculator to find the x -intercept(s).
 • Solution is $x = \frac{1}{2}$.
3. a) $x = -5$ b) $x = -\frac{3}{5}$ or $x = \frac{5}{3}$ c) $x = -14$ d) $x = \frac{17}{5}$ or $x = 31$
4. a) -15 or 5 b) -1 or $\frac{5}{3}$ c) no solution
 d) $\frac{1}{10}$ e) no solution f) -14
5. a) $\frac{17}{5}, 31$ b) 1 c) $\pm 4, \pm 6$ d) $-9, -7, -3, -1$
6. a) $-2.3, -0.7, 2.7, 4.3$ b) $-3.8, 1.0, 2.0, 6.8$
7. $1 \pm \sqrt{3}, 1 \pm \sqrt{11}$ 8. D 9.

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Group Work

$x = -5, 1$ (The method is covered in the next lesson.)