Absolute Value Functions and Reciprocal Functions Lesson #1:
Absolute Value Functions

The Absolute Value of a Number

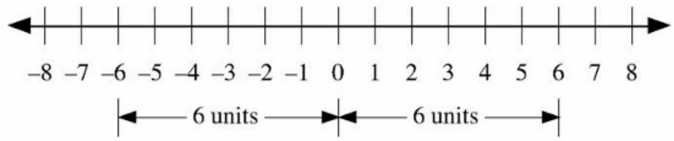
The **absolute value** of a real number can be defined as the principal square root of the square of the number.

e.g. the absolute value of $6 = \sqrt{(6)^2} = \sqrt{36} = 6$

the absolute value of $-6 = \sqrt{(-6)^2} = \sqrt{36} = 6$

For a real number, a, the absolute value of a is written $|a|$. e.g. $|6| = 6$ and $|-6| = 6$.

The absolute value of a real number can be regarded as the distance of the number from zero on a number line.



The absolute value of a number will never be negative.

Evaluate:

a) $|3|$ **b)** $|-3|$ **c)** $-|8|$ **d)** $-|-8|$

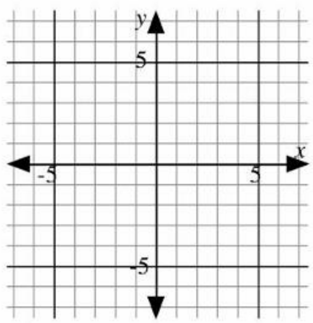
e) $|-7| + |7|$ **f)** $|1 - 5|$ **g)** $-|-\sqrt{81}\,|$

Complete Assignment Questions #1 - #3

Investigating the Function $f(x) = |x|$

1. Consider the functions $g(x) = x$, and $h(x) = -x$, whose graphs respectively have equations $y = x$ and $y = -x$.

Sketch and label each graph on the grid below.

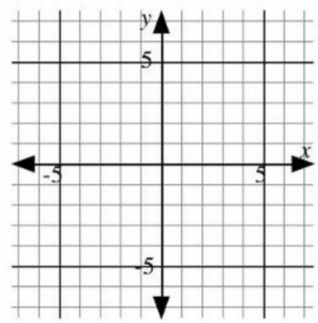


2. Consider the function $f(x) = |x|$, whose graph has the equation $y = |x|$.

a) Complete the table of values.

b) Plot the points on the grid and join the points.

x	y
–5	
–4	
–3	
–2	
–1	
0	
1	
2	
3	
4	
5	



3. Explain the similarities and differences between the graphs on the two grids above.

4. The graph of $y = |x|$ contains two straight lines.

a) State the equation of the line in quadrant 1.

b) State the equation of the line in quadrant 2.

5. We can see from #4 that the equation $y = |x|$ can be written in two **pieces** with <u>different domains for each piece</u>.

Complete the following to write the absolute value function $f(x) = |x|$ as a **piecewise function**;

$$f(x) = \begin{cases} & \text{if } x \geq 0 \\ & \text{if } x < 0 \end{cases}$$

Recall from the definition of the absolute value of a number on the previous page that the absolute value function $f(x) = |x|$ could be written as $f(x) = \sqrt{x^2}$.

Defining The Absolute Value Function $f(x) = |x|$

The absolute value function $f(x) = |x|$ can be defined as:

$$f(x) = |x| = \sqrt{x^2} = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Notice that when x is a positive number, $|x| = x$, and when x is a negative number, $|x| = -x$.

Investigating the Function $f(x) = |x - 3|$

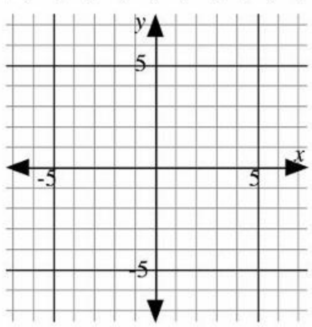
1. Consider the functions $g(x) = x - 3$, and $h(x) = -(x - 3)$, whose graphs respectively have equations $y = x - 3$ and $y = -(x - 3)$.

a) Complete the tables of values.

b) Plot the points on the grid, join the points, and extend the graphs.

x	-4	-3	-2	-1	0	1	2	3	4	5
$y = x - 3$										

x	-4	-3	-2	-1	0	1	2	3	4	5
$y = -(x - 3)$										

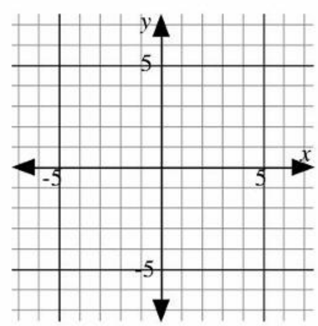


2. Consider the function $f(x) = |x - 3|$, whose graph has the equation $y = |x - 3|$.

a) Complete the table of values.

b) Plot the points on the grid, join the points, and extend the graph.

x	y
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	



3. The function $f(x) = |x - 3|$ is written below as a piecewise function, but the domain for each piece has been omitted.

Use the graph of $y = |x - 3|$ to determine the domain for each piece, and complete the piecewise function form of $f(x) = |x - 3|$ shown below.

$$f(x) = \begin{cases} x - 3 & \text{if} \\ -(x - 3) & \text{if} \end{cases}$$

4. Explain how the equation of the second piece can be determined from the equation of the first piece.

5. Explain algebraically, without using a graph, how the domain for each piece could be determined.

- Every absolute value function can be defined in pieces.
- The absolute value of a quantity will always be the same quantity if the quantity is positive and the opposite quantity if the quantity is negative.
- Writing an absolute value function in piecewise form is an integral part of determining the solution to absolute value equations or inequalities.

a) Express $f(x) = |3x + 2|$ as a piecewise function.

b) Write the absolute value expression $|4 - x|$ in piecewise form.

Complete Assignment Questions #4 - #11

Assignment

1. Evaluate:

a) $|8|$ **b)** $|-8|$ **c)** $-|7|$ **d)** $-|-7|$

e) $|-2| - |2|$ **f)** $|-23| + |15|$ **g)** $|16 - 25|$ **h)** $|12 - 22| - 11$

2. Evaluate:

a) $|3 - 9|$ **b)** $|3| - |9|$ **c)** $||3| - |9||$ **d)** $-|-\sqrt{81}|$

e) $-|\sqrt[3]{27}|$ **f)** $|-\sqrt[3]{27}|$ **g)** $|\sqrt[3]{-27}|$ **h)** $|-\sqrt[3]{-27}|$

3. Which of the following statements are true and which are false?

a) $|-7| = |7|$ **b)** $|3 - 6| = -3$ **c)** $|2| - |4| = |-2|$ **d)** $||5| - |-32|| = 27$

4. Write the following absolute value functions as piecewise functions.

a) $f(x) = |x|$ **b)** $g(x) = |x + 1|$

c) $f(x) = |x - 2|$ **d)** $g(x) = |3 - x|$

5. Write the following absolute value expressions in piecewise form.

a) $|2x + 1|$ **b)** $|4x - 1|$

c) $|2 + x|$ **d)** $|4 - 2x|$

6. Decide whether each statement is true or false.

a) $|x| = x$ if $x > 0$ **b)** $|x| = -x$ if $x < 0$

7. Given that the absolute value of a number is a positive quantity, explain why "$|x| = -x$ if $x < 0$" is a true statement when it appears that the right hand side of the equation is a negative quantity.

8. Which of the following statements are true and which are false?

a) $|-x| = x$, if $x < 0$ **b)** $|-x| = -x$, if $x \geq 0$

c) $|2x - 1| = 2x - 1$, if $x < \frac{1}{2}$ **d)** $|3x + 4| = -3x - 4$, if $x < -\frac{4}{3}$

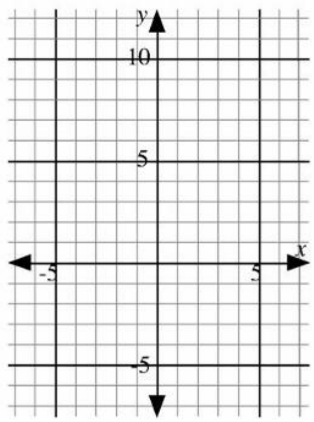
e) $|2 - 5x| = 2 - 5x$, if $x \geq \frac{2}{5}$ **f)** $|x - 7| = -x - 7$, if $x < 7$

9. a) Consider the function $g(x) = x^2 - 4$, whose graph has the equation $y = x^2 - 4$.

i) Complete the tables of values.

ii) Plot the points on the grid, join the points, and extend the graph.

x	y
–4	
–3	
–2	
–1	
0	
1	
2	
3	
4	

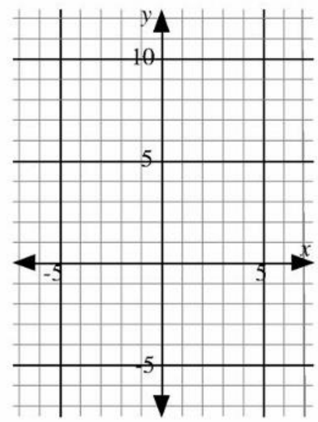


b) Consider the function $f(x) = |x^2 - 4|$, whose graph has the equation $y = |x^2 - 4|$.

i) Complete the table of values.

ii) Plot the points on the grid, join the points, and extend the graph.

x	y
–4	
–3	
–2	
–1	
0	
1	
2	
3	
4	



c) The function $f(x) = |x^2 - 4|$ can be written as a piecewise function in three pieces. Complete the piecewise function for $f(x) = |x^2 - 4|$ shown.

$$f(x) = \begin{cases} \qquad & \text{if} \quad x < -2 \\ \qquad & \text{if} \quad -2 \le x \le 2 \\ \qquad & \text{if} \quad x > 2 \end{cases}$$

d) Write the following absolute value functions as piecewise functions.

i) $f(x) = |x^2 - 25|$

ii) $g(x) = |36 - x^2|$

10. Explain why the function $f(x) = |x^2 + 4|$ can be written, without absolute value symbols, as only a single piece.

Multiple Choice

11. Which of the following is false?

A. $|x-9| = x-9$ if $x \geq 9$

B. $|9-x| = 9-x$ if $x < 9$

C. $|9-x^2| = 9-x^2$ if $-3 \leq x \leq 3$

D. $|x^2-9| = x^2-9$ if $-3 \leq x \leq 3$

Answer Key

1. a) 8 b) 8 c) –7 d) –7 e) 0 f) 38 g) 9 h) –1

2. a) 6 b) –6 c) 6 d) –9 e) –3 f) 3 g) 3 h) 3

3. a) T b) F c) F d) T

4. a) $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$ b) $g(x) = \begin{cases} x+1 & \text{if } x \geq -1 \\ -x-1 & \text{if } x < -1 \end{cases}$

c) $f(x) = \begin{cases} x-2 & \text{if } x \geq 2 \\ -x+2 & \text{if } x < 2 \end{cases}$ d) $g(x) = \begin{cases} 3-x & \text{if } x \leq 3 \\ -3+x & \text{if } x > 3 \end{cases}$

5. a) $|2x+1| = \begin{cases} 2x+1 & \text{if } x \geq -\frac{1}{2} \\ -2x-1 & \text{if } x < -\frac{1}{2} \end{cases}$ b) $|4x-1| = \begin{cases} 4x-1 & \text{if } x \geq \frac{1}{4} \\ -4x+1 & \text{if } x < \frac{1}{4} \end{cases}$

c) $|2+x| = \begin{cases} 2+x & \text{if } x \geq -2 \\ -2-x & \text{if } x < -2 \end{cases}$ d) $|4-2x| = \begin{cases} 4-2x & \text{if } x \leq 2 \\ -4+2x & \text{if } x > 2 \end{cases}$

6. a) T b) T

7. Although $-x$ might appear to be a negative quantity, it is in fact a positive quantity if x is negative.

8. a) F b) F c) F d) T e) F f) F

9. c) $f(x) = \begin{cases} x^2-4 & \text{if } x < -2 \\ -x^2+4 & \text{if } -2 \leq x \leq 2 \\ x^2-4 & \text{if } x > 2 \end{cases}$

d) i) $f(x) = \begin{cases} x^2-25 & \text{if } x < -5 \\ -x^2+25 & \text{if } -5 \leq x \leq 5 \\ x^2-25 & \text{if } x > 5 \end{cases}$ ii) $g(x) = \begin{cases} -36+x^2 & \text{if } x < -6 \\ 36-x^2 & \text{if } -6 \leq x \leq 6 \\ -36+x^2 & \text{if } x > 6 \end{cases}$

10. Since x^2+4 is positive for all values of x, $|x^2+4|$ can be written as x^2+4 for $x \in R$.

11. D