Absolute Value Functions and Reciprocal Functions Lesson #5:
Reciprocal Functions

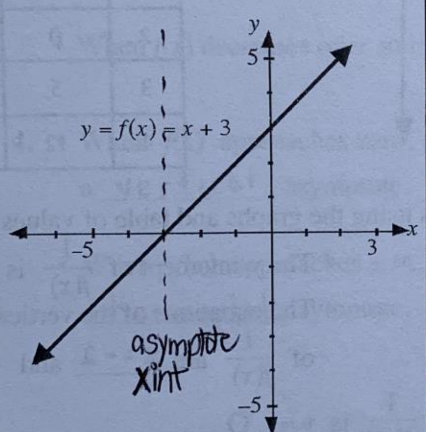
Reciprocal Functions

The reciprocal function of $f(x)$ is $\dfrac{1}{f(x)}$. For example, if a function has the equation $y = x^2 - 5$, the reciprocal function has equation $y = \dfrac{1}{x^2 - 5}$.

Exploring a Reciprocal Function

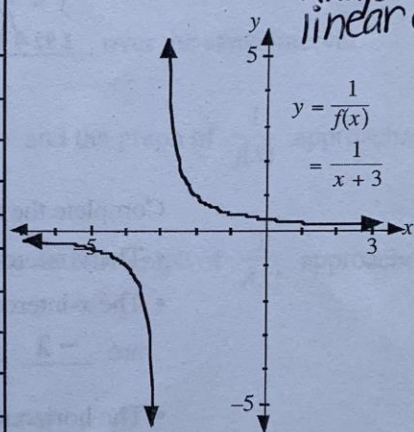
1.a) Consider the function $f(x)$ with equation $y = x + 3$.
Write the equation of the reciprocal function. $y = \dfrac{1}{x+3}$

b) The graphs of $y = f(x)$ and $y = \dfrac{1}{f(x)}$ and a partial table of values are shown.



x	$y = f(x)$	$y = \frac{1}{f(x)}$
−6	−3	−⅓
−5	−2	−½
−4	−1	−1
−3	0	Undefined
−2	1	1
−1	2	½
0	3	⅓
1	4	¼
2	5	⅕
3	6	⅙

Always for linear eq^Ns:



Complete the following statements using the graphs and table of values.

- The y-intercept of $f(x)$ is <u>3</u>.
- The y-intercept of $\dfrac{1}{f(x)}$ is <u>$\frac{1}{3}$</u>.
- The x-intercept of $f(x)$ is <u>−3</u>.
- The equation of the vertical asymptote of $\dfrac{1}{f(x)}$ is $x =$ <u>−3</u>.
- State the coordinates of the two points which appear on **both** the graph of $y = f(x)$ and the graph of $y = \dfrac{1}{f(x)}$. These points are called **invariant points**. (−4, −1) (−2, 1)
  points don't change
- The horizontal asymptote of $y = \dfrac{1}{f(x)}$ is $y =$ <u>0</u>.

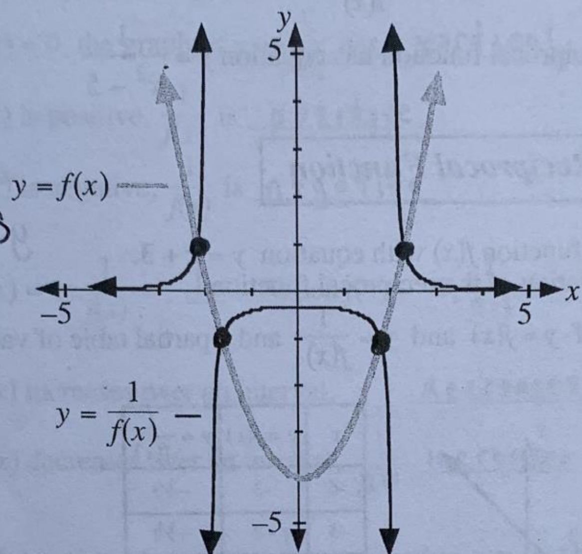
c) Complete the following:

- When $f(x) = 0$, the graph of $y = \dfrac{1}{f(x)}$ has a <u>vertical</u> asymptote.
- As $f(x)$ approaches $\pm\infty$, (positive or negative infinity), the graph of $y = \dfrac{1}{f(x)}$ approaches closer to the <u>horizontal</u> asymptote.

2.a) Consider the function $f(x)$ with equation $y = x^2 - 4$. Write the equation of the reciprocal function. $y = \dfrac{1}{x^2-4}$

b) The graphs of $y = f(x)$ and $y = \dfrac{1}{f(x)}$ and a partial table of values are shown.

ALWAYS FOR QUADRATICS



x	$y = f(x)$	$y = \dfrac{1}{f(x)}$
-4	12	1/12
-3	5	1/5
−2	0	Undefined
−1	−3	−1/3
0	−4	−1/4
1	−3	−1/3
2	0	Undefined
3	5	1/5
4	12	1/12

Complete the following statements using the graphs and table of values.

- The y-intercept of $f(x)$ is -4.
- The y-intercept of $\dfrac{1}{f(x)}$ is $-\dfrac{1}{4}$.
- The x-intercepts of $f(x)$ are -2 and 2.
- The equations of the vertical asymptotes of $\dfrac{1}{f(x)}$ are $x = -2$ and $x = 2$.
- The horizontal asymptote of $y = \dfrac{1}{f(x)}$ is $y = 0$.

c) In example 1 of the previous page we were able to determine the invariant points of the graphs of $f(x)$ and $\dfrac{1}{f(x)}$ by using the table of values for $y = \pm 1$. Explain why we can use the lines $y = \pm 1$ to find the invariant points of the graphs of $f(x)$ and $\dfrac{1}{f(x)}$. Mark these points on the above sketch.

the reciprocal of 1 is $\frac{1}{1} = 1$ When $y = \pm 1$, $\frac{1}{y} = \pm 1$

the reciprocal of −1 is $\frac{1}{-1} = -1$

d) Complete the following:

- When $f(x) = 0$, the graph of $y = \dfrac{1}{f(x)}$ has vertical asymptotes.
- As $f(x)$ approaches $\pm\infty$, the graph of $y = \dfrac{1}{f(x)}$ approaches closer to the horizontal asymptote with equation $y = 0$.

Properties of Reciprocal Transformations

Complete the following statements based on the exploration on the previous two pages.

1. • When $f(x) = 0$, the graph of $y = \frac{1}{f(x)}$ has a vertical asymptote.
   • When $f(x)$ is positive, $\frac{1}{f(x)}$ is positive.
   • When $f(x)$ is negative, $\frac{1}{f(x)}$ is negative.

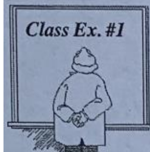
2. • When $f(x) = 1$, $\frac{1}{f(x)} =$ 1. When $f(x) = -1$, $\frac{1}{f(x)} =$ -1.

3. • When $f(x)$ increases over an interval, $\frac{1}{f(x)}$ decreases over the same interval.
   • When $f(x)$ decreases over an interval, $\frac{1}{f(x)}$ increases over the same interval.

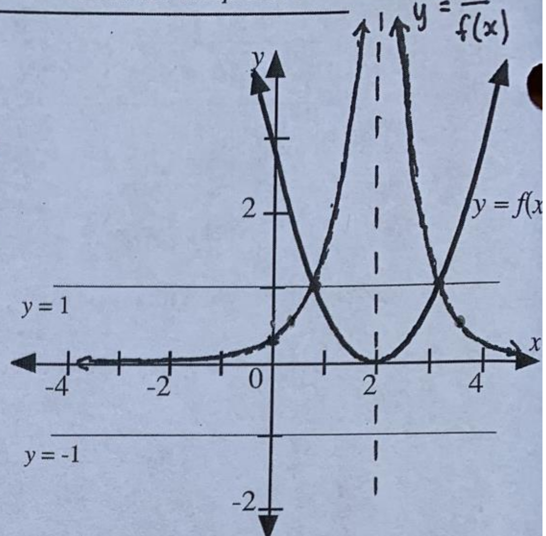
4. • When $f(x)$ approaches zero, $\frac{1}{f(x)}$ approaches $\pm\infty$ and the graph of $\frac{1}{f(x)}$ approaches a vertical asymptote.
   • When $f(x)$ approaches $\pm\infty$, $\frac{1}{f(x)}$ approaches zero and the graph of $\frac{1}{f(x)}$ approaches a horizontal asymptote.

Suggestions for Sketching the Graph of a Reciprocal Function

1. Zeros of the original function become vertical asymptotes of the reciprocal function.
2. Mark the **invariant points** where $y = 1$ and $y = -1$.
3. The y-intercept of the reciprocal graph is the reciprocal of the y-intercept on the original graph.
4. Points where $y = 2$ on the original graph become points where $y = \frac{1}{2}$ on the reciprocal graph, etc.
5. Complete the reciprocal graph based on the information above.


Class Ex. #1

The graph of $y = f(x)$ is shown.
The x-intercept is 2 and the y-intercept is 3.



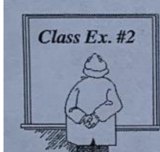
a) Use the suggestions on the previous page to sketch the graph of $y = \frac{1}{f(x)}$.
The lines with equations $y = 1$ and $y = -1$ have been provided as a guide.

b) State the y-intercept of the graph of $y = \frac{1}{f(x)}$. $\frac{1}{3}$

c) Given that the graph of $f(x)$ has equation $y = \frac{3}{4}(x-2)^2$, state the equation of $y = \frac{1}{f(x)}$.

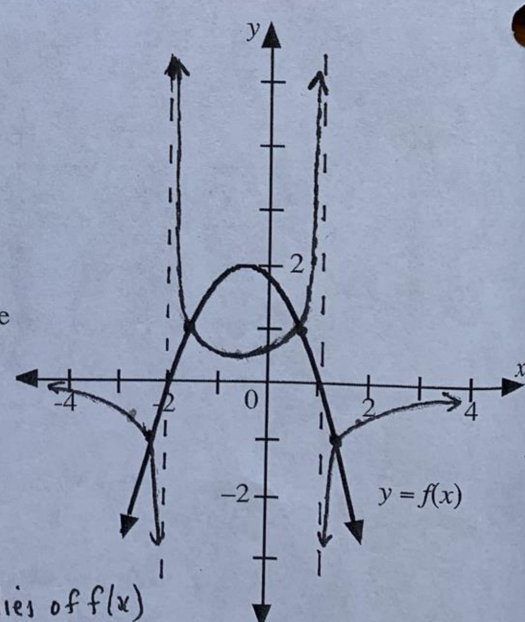
$y = \frac{1}{\frac{3}{4}(x-2)^2}$ or $y = \frac{4}{3(x-2)^2}$

d) Use a graphing calculator to verify the graph drawn in a).


Class Ex. #2

The graph of the quadratic function $y = f(x)$ is shown.

The x-intercepts of the graph are integers, and the maximum value of f is 2.



a) Sketch the graph of $y = \frac{1}{f(x)}$.

b) State the equations of the asymptotes of the graph of $y = \frac{1}{f(x)}$.

$y = 0$ $x = -2$ $x = 1$

c) The point $\left(a, \frac{1}{2}\right)$ lies on the graph of $\frac{1}{f(x)}$.
State the value of a.

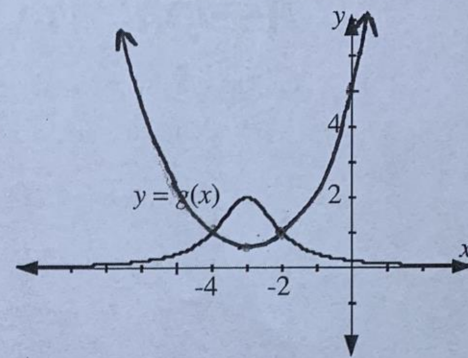
If $(a, \frac{1}{2})$ lies on $\frac{1}{f(x)}$ then $(a, 2)$ lies of $f(x)$

x-coordinate of vertex $= \frac{-2+1}{2} = -\frac{1}{2}$ $\underline{\underline{a = -\frac{1}{2}}}$

Complete Assignment Questions #1 - #8

Class Ex. #3

The graph of $g(x) = \dfrac{1}{f(x)}$ is shown.
The maximum point of $g(x)$ is at $(-3, 2)$ and the y-intercept of $g(x)$ is $\dfrac{1}{5}$.



a) Given that $f(x)$ is a quadratic function, sketch the graph of $y = f(x)$ on the grid and state the coordinates of the minimum point.

yint of $f(x)$ is 5
minimum point of $f(x)$ is at $(-3, \frac{1}{2})$
invariant points where $y = 1$
by symmetry the point $(-6, 5)$ is on the graph of f.

b) $f(x)$ can be written in the form $y = a(x - p)^2 + q$. Determine the values of a, p, and q.

vertex of f $(-3, \frac{1}{2})$ $\quad p = -3 \quad q = \frac{1}{2}$

$y = a(x+3)^2 + \frac{1}{2}$

$(0, 5) \rightarrow \quad 5 = a(0+3)^2 + \frac{1}{2}$

$5 = 9a + \frac{1}{2}$

$4.5 = 9a$

$a = \frac{1}{2}$

$y = \frac{1}{2}(x+3)^2 + \frac{1}{2}$

$a = \frac{1}{2}, \ p = -3, \ q = \frac{1}{2}$