Absolute Value Functions and Reciprocal Functions Lesson #5: Reciprocal Functions

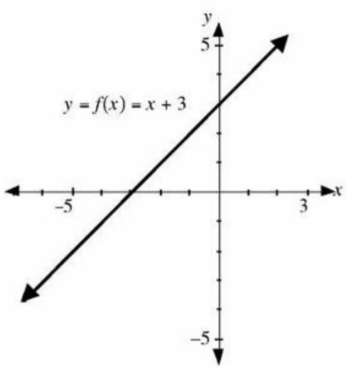
Reciprocal Functions

The reciprocal function of $f(x)$ is $\frac{1}{f(x)}$. For example, if a function has the equation $y = x^2 - 5$, the reciprocal function has equation $y = \frac{1}{x^2 - 5}$.

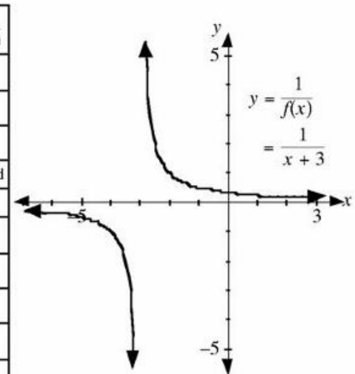
Exploring a Reciprocal Function

1.a) Consider the function $f(x)$ with equation $y = x + 3$.
Write the equation of the reciprocal function.

b) The graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$ and a partial table of values are shown.



x	$y = f(x)$	$y = \frac{1}{f(x)}$
–6	–3	$-\frac{1}{3}$
–5	–2	$-\frac{1}{2}$
–4	–1	–1
–3	0	Undefined
–2	1	1
-1	2	$\frac{1}{2}$
0	3	$\frac{1}{3}$
1	4	$\frac{1}{4}$
2	5	$\frac{1}{5}$
3	6	$\frac{1}{6}$



Complete the following statements using the graphs and table of values.

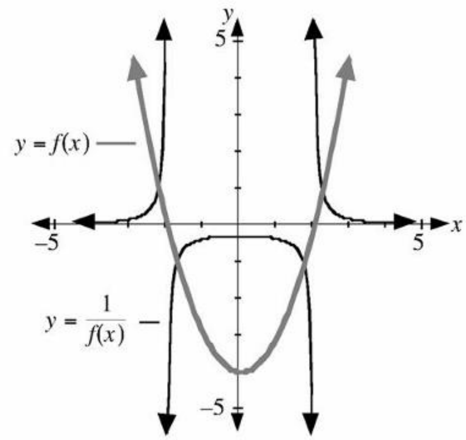
- The y-intercept of $f(x)$ is _____ .
- The y-intercept of $\frac{1}{f(x)}$ is _____ .
- The x-intercept of $f(x)$ is _____ .
- The equation of the vertical asymptote of $\frac{1}{f(x)}$ is $x =$ _____ .
- State the coordinates of the two points which appear on **both** the graph of $y = f(x)$ and the graph of $y = \frac{1}{f(x)}$. These points are called **invariant points**.
- The horizontal asymptote of $y = \frac{1}{f(x)}$ is $y =$ _____ .

c) Complete the following:

- When $f(x) = 0$, the graph of $y = \frac{1}{f(x)}$ has a _______________ asymptote.
- As $f(x)$ approaches $\pm\infty$, (positive or negative infinity), the graph of $y = \frac{1}{f(x)}$ approaches closer to the _______________ asymptote.

2.a) Consider the function $f(x)$ with equation $y = x^2 - 4$.
Write the equation of the reciprocal function.

b) The graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$ and a partial table of values are shown.



x	$y = f(x)$	$y = \frac{1}{f(x)}$
-4	12	1/12
-3	5	1/5
−2	0	Undefined
−1	−3	−1/3
0	−4	−1/4
1	−3	−1/3
2	0	Undefined
3	5	1/5
4	12	1/12

Complete the following statements using the graphs and table of values.

- The y-intercept of $f(x)$ is _____ .
- The y-intercept of $\frac{1}{f(x)}$ is _____ .
- The x-intercepts of $f(x)$ are _____ and _____ .
- The equations of the vertical asymptotes of $\frac{1}{f(x)}$ are _____ and _____ .
- The horizontal asymptote of $y = \frac{1}{f(x)}$ is $y =$ _____ .

c) In example 1 of the previous page we were able to determine the invariant points of the graphs of $f(x)$ and $\frac{1}{f(x)}$ by using the table of values for $y = \pm 1$. Explain why we can use the lines $y = \pm 1$ to find the invariant points of the graphs of $f(x)$ and $\frac{1}{f(x)}$. Mark these points on the above sketch.

d) Complete the following:

- When $f(x) =$ ____, the graph of $y = \frac{1}{f(x)}$ has vertical asymptotes.
- As $f(x)$ approaches ___________ , the graph of $y = \frac{1}{f(x)}$ approaches closer to the horizontal asymptote with equation $y =$ ____ .

Properties of Reciprocal Transformations

Complete the following statements based on the exploration on the previous two pages.

1. • When $f(x) = 0$, the graph of $y = \frac{1}{f(x)}$ has a ____________ ____________ .

• When $f(x)$ is positive, $\frac{1}{f(x)}$ is ____________ .

• When $f(x)$ is negative, $\frac{1}{f(x)}$ is ____________ .

2. • When $f(x) = 1$, $\frac{1}{f(x)}$ = ____ . When $f(x) = -1$, $\frac{1}{f(x)}$ = ____ .

3. • When $f(x)$ increases over an interval, $\frac{1}{f(x)}$ ____________ over the same interval.

• When $f(x)$ decreases over an interval, $\frac{1}{f(x)}$ ____________ over the same interval.

4. • When $f(x)$ approaches zero, $\frac{1}{f(x)}$ approaches $\pm\infty$ and the graph of $\frac{1}{f(x)}$ approaches a ____________ asymptote.

• When $f(x)$ approaches $\pm\infty$, $\frac{1}{f(x)}$ approaches zero and the graph of $\frac{1}{f(x)}$ approaches a ____________ asymptote.

Suggestions for Sketching the Graph of a Reciprocal Function

1. Zeros of the original function become vertical asymptotes of the reciprocal function.

2. Mark the **invariant points** where $y = 1$ and $y = -1$.

3. The y-intercept of the reciprocal graph is the reciprocal of the y-intercept on the original graph.

4. Points where $y = 2$ on the original graph become points where $y = \frac{1}{2}$ on the reciprocal graph, etc.

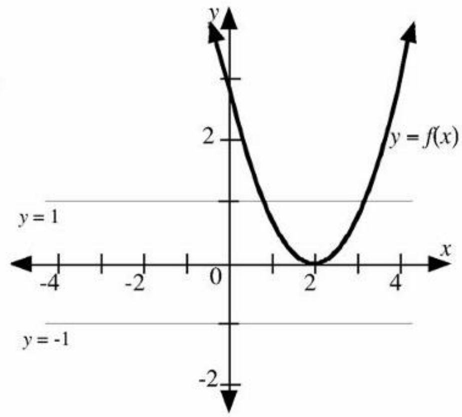
5. Complete the reciprocal graph based on the information above.

The graph of $y = f(x)$ is shown.
The x-intercept is 2 and the y-intercept is 3.

a) Use the suggestions on the previous page to sketch the graph of $y = \frac{1}{f(x)}$.
The lines with equations $y = 1$ and $y = -1$ have been provided as a guide.

b) State the y-intercept of the graph of $y = \frac{1}{f(x)}$.

c) Given that the graph of $f(x)$ has equation $y = \frac{3}{4}(x-2)^2$, state the equation of $y = \frac{1}{f(x)}$.



d) Use a graphing calculator to verify the graph drawn in a).

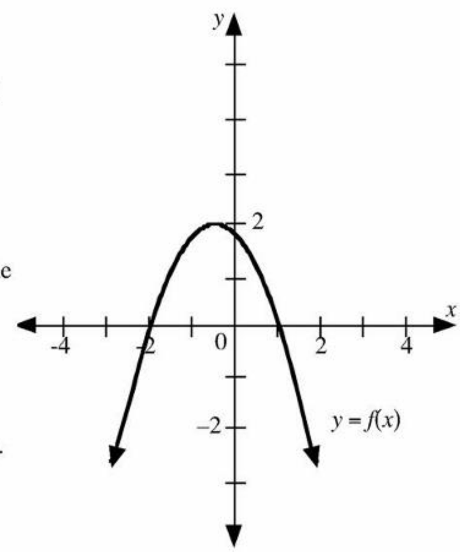
The graph of the quadratic function $y = f(x)$ is shown.

The x-intercepts of the graph are integers, and the maximum value of f is 2.

a) Sketch the graph of $y = \frac{1}{f(x)}$.

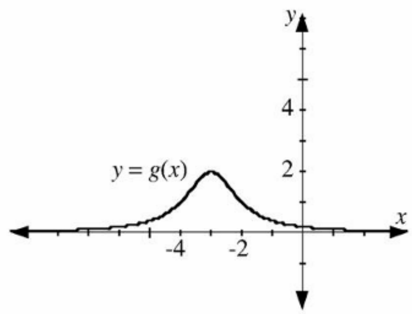
b) State the equations of the asymptotes of the graph of $y = \frac{1}{f(x)}$.

c) The point $\left(a, \frac{1}{2}\right)$ lies on the graph of $\frac{1}{f(x)}$.
State the value of a.



Complete Assignment Questions #1 - #8

The graph of $g(x) = \dfrac{1}{f(x)}$ is shown.
The maximum point of $g(x)$ is at $(-3, 2)$ and the y-intercept of $g(x)$ is $\dfrac{1}{5}$.



a) Given that $f(x)$ is a quadratic function, sketch the graph of $y = f(x)$ on the grid and state the coordinates of the minimum point.

b) $f(x)$ can be written in the form $y = a(x - p)^2 + q$. Determine the values of a, p, and q.

Complete Assignment Questions #9 - #13

Assignment

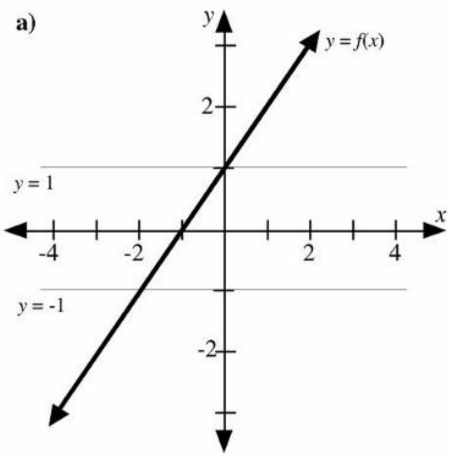
1. The graph of $y = f(x)$ is given. In each case:

i) sketch the graph of $y = \dfrac{1}{f(x)}$

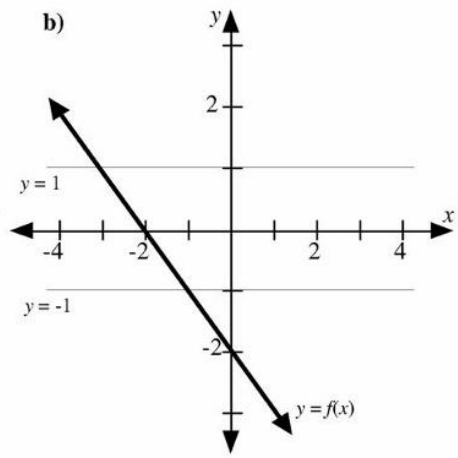
ii) write the equation of the vertical asymptote of the graph of $y = \dfrac{1}{f(x)}$

iii) state the y-intercept of the graph of $y = \dfrac{1}{f(x)}$

a)

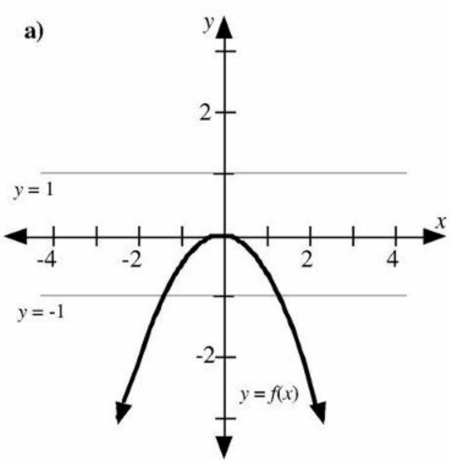


b)

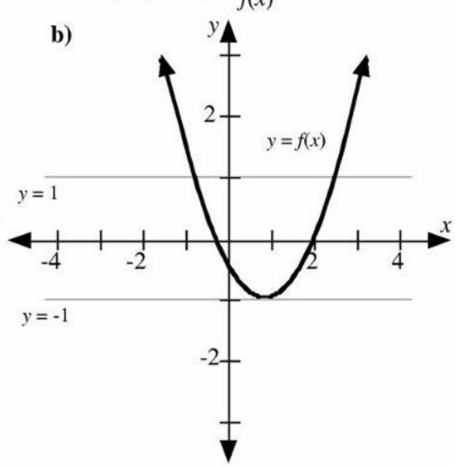


2. The graph of $y = f(x)$ is given. In each case sketch the graph of $y = \dfrac{1}{f(x)}$.

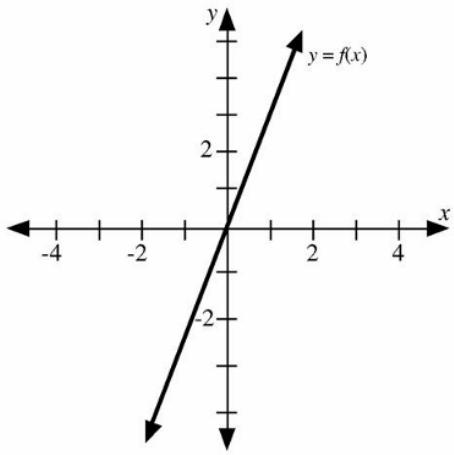
a)

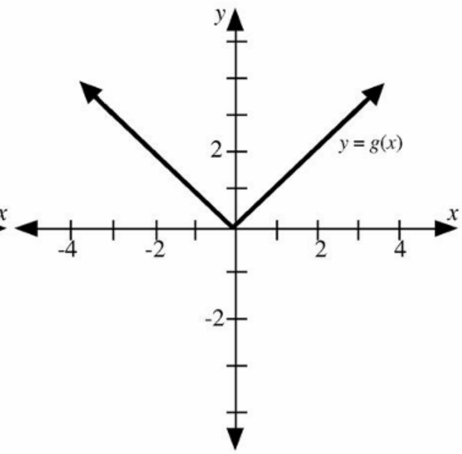


b)



3. The graphs of $y = f(x)$ and $y = g(x)$ are shown. Sketch the graphs of $y = \frac{1}{f(x)}$ and $y = \frac{1}{g(x)}$ and explain why neither graph has a y-intercept.



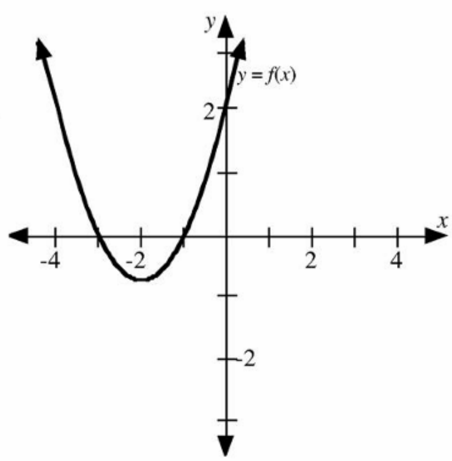


4. The graph of $y = f(x)$ is shown.
The y-intercept of the graph is 2.

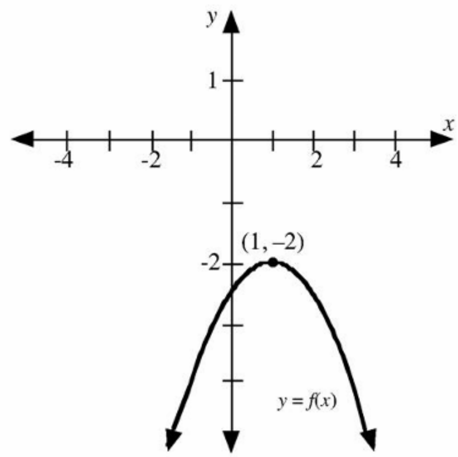
a) State the y-intercept of the graph of $\frac{1}{f(x)}$.

b) If the graph of $y = \frac{1}{f(x)}$ passes through the point $(-2, -1.25)$, sketch the graph of $\frac{1}{f(x)}$ on the grid.

c) Determine the minimum value of f.



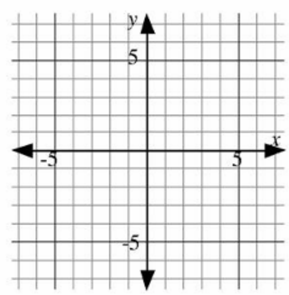
5. a) Use the information given to sketch the graph of $y = \frac{1}{f(x)}$.



b) Explain why the graph of $\frac{1}{f(x)}$ has no vertical asymptotes.

c) Explain why the graph of $y = \frac{1}{f(x)}$ has no points in quadrants 1 and 2.

6. A function $f(x)$ has equation $y = x + 3$.

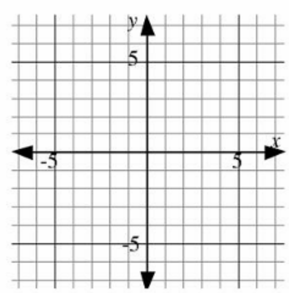


a) Write the equation of the reciprocal function.

b) Use a graphing calculator to sketch the graph of $y = f(x)$ and $y = \frac{1}{f(x)}$ on the grid.

c) State the coordinates of the invariant points in b).

7. A function $f(x)$ has equation $y = 5 - x^2$.



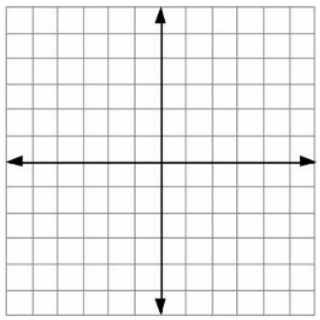
a) Write the equation of the reciprocal function.

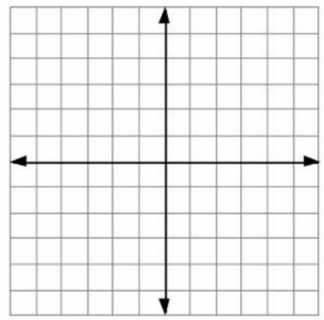
b) Use a graphing calculator to sketch the graph of $y = f(x)$ and $y = \frac{1}{f(x)}$ on the grid.

c) State the coordinates of the invariant points in quadrant 1.

8. a) Sketch the graphs of:

i) $y = x$ **ii)** $y = \frac{1}{x}$

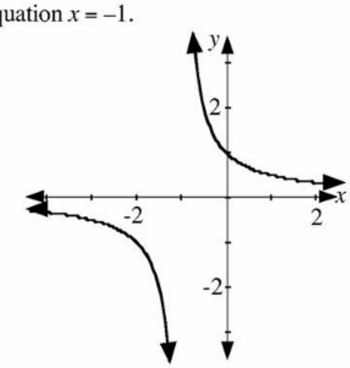




b) State the coordinates of the invariant points on the graphs above.

9. The graph of $g(x) = \frac{1}{f(x)}$, where $f(x)$ is a linear function, is shown. The graph of $y = g(x)$ has a y-intercept of 1 and a vertical asymptote with equation $x = -1$.

a) Describe a strategy for sketching the graph of $y = f(x)$ on the grid.

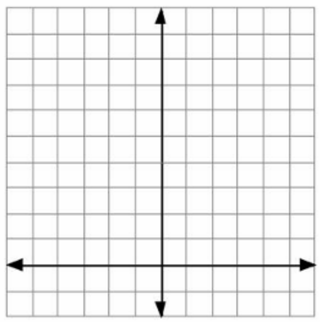


b) Sketch the graph of $y = f(x)$ on the grid.

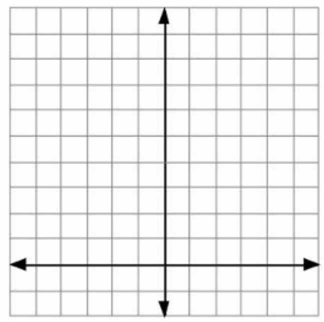
c) If the function $f(x)$ has equation $y = ax + b$, determine the values of a and b.

10. a) Sketch the graphs of:

i) $y = x^2$

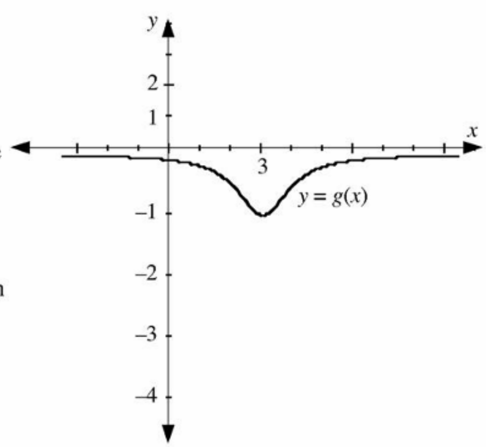


ii) $y = \frac{1}{x^2}$



b) State the coordinates of the invariant points on the graphs above.

11. Consider the quadratic function $f(x) = ax^2 + bx + c$.
The graph of $g(x) = \frac{1}{f(x)}$ is shown.



a) If the y-intercept of $g(x)$ is $-\frac{1}{3}$, state the y-intercept of $f(x)$.

b) Given that the minimum point of $g(x)$ is at $(3, -1)$, sketch the graph of $y = f(x)$ on the grid.

c) State the maximum point on the graph of $y = f(x)$.

d) Express $f(x)$ in the form $f(x) = a(x - p)^2 + q$.

e) Express $f(x)$ in the form $f(x) = ax^2 + bx + c$.

Use the following information to answer the next question.

A student made the following statements about reciprocal functions.

Statement 1: Vertical asymptotes on the graph of $y = \frac{1}{f(x)}$ are drawn through the x-intercepts on the graph of $y = f(x)$.

Statement 2: If the range of the graph of $y = f(x)$ is $y > 0, y \in R$, then the range of $y = \frac{1}{f(x)}$ is $y < 0, y \in R$.

Statement 3: The invariant points of the graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$ are the intersection points of the lines $y = \pm 1$ and the graphs of $f(x)$ and $\frac{1}{f(x)}$.

Multiple Choice **12.** Which of the student's statements are true?

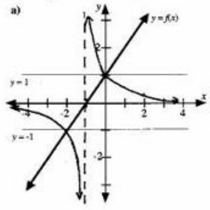
A. 1 and 2 only **B.** 1 and 3 only **C.** 2 and 3 only **D.** 1, 2, and 3

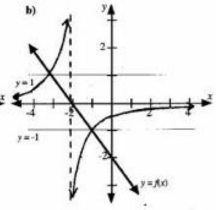
Numerical Response **13.** The graph of $y = f(x)$ passes through the point $(4, 6)$. The graph of $y = \frac{1}{f(x)}$ passes through the point $(4, p)$. The value of p to the nearest hundredth is ______ .

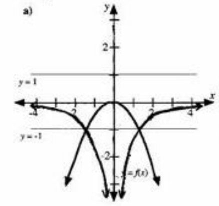
(Record your answer in the numerical response box from left to right)

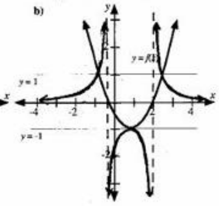
Answer Key

1. a) See below **b)** See below **2.a)** See below **b)** See below



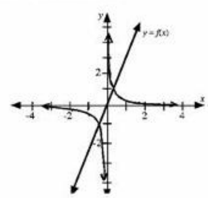


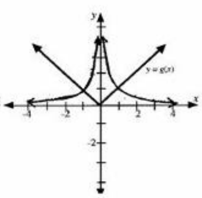




ii) $x = -1$ **iii)** 1 **ii)** $x = -2$ **iii)** $-\frac{1}{2}$

3. See below.



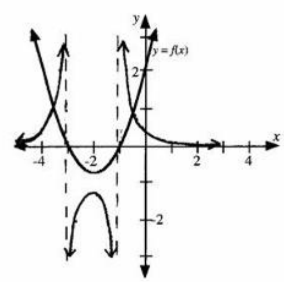


Both $y = f(x)$ and $y = g(x)$ have a y-intercept of 0 and an x-intercept of 0.

The graphs of $y = \dfrac{1}{f(x)}$ and $y = \dfrac{1}{g(x)}$ have asymptotes with equation $x = 0$, and so do not have any y-intercepts.

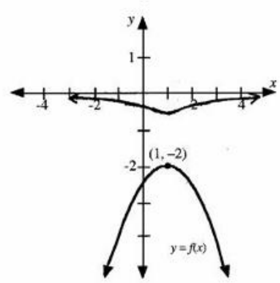
4. a) $\dfrac{1}{2}$

b) See graph at the right



c) $\left(-2, -\frac{5}{4}\right)$ lies on $y = \frac{1}{f(x)}$ so $\left(-2, -\frac{4}{5}\right)$ lies on $y = f(x)$.
minimum value $f = -\frac{4}{5}$ or -0.8

5. a) See graph at the right.



b) The graph of $y = f(x)$ has no x-intercepts, so the graph of $y = f(x)$ has no vertical asymptotes.

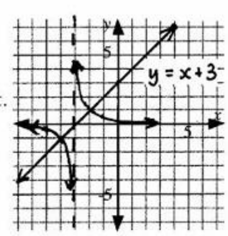
c) Since $f(x)$ is always negative, $\dfrac{1}{f(x)}$ is always negative

The graph of $y = \dfrac{1}{f(x)}$ has no points in quadrants 1 and 2.

6. a) $y = \dfrac{1}{x + 3}$

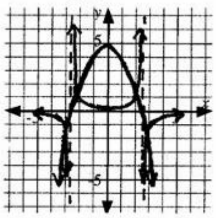
b) See graph at the right.

c) (−2, 1), (−4, −1)



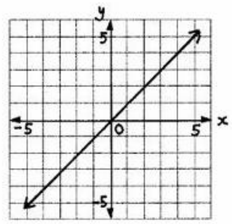
7. a) $y = \dfrac{1}{5 - x^2}$

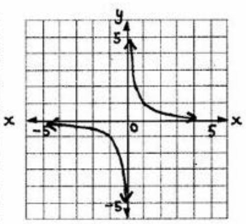
b) See graph below



c) (2, 1)

8. a) i) See graph below **ii)** See graph below

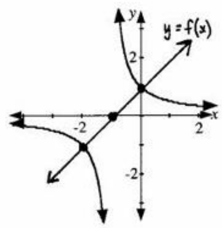




b) For both graphs, the invariant points are (1, 1) and (–1, –1).

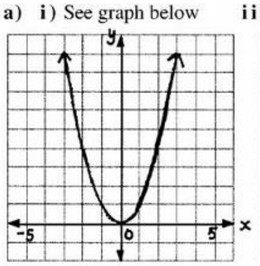
9. a) The asymptote of $y = g(x)$ becomes a zero of $y = f(x)$.

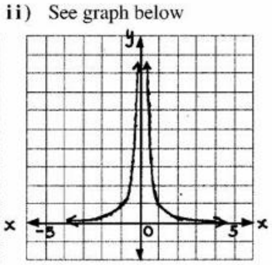
b) See graph to the right



c) Points (0, 1) and (–1, 0).
Slope = 1, so $a = 1$
y-intercept = 1, so $b = 1$
$y = x + 1$.

10. a) i) See graph below **ii)** See graph below





b) For both graphs, the invariant points are (1, 1) and (–1, 1).

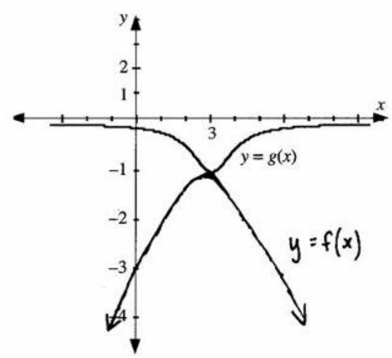
11. a) –3

b) See graph at the right.

c) (3, –1)

d) $f(x) = -\dfrac{2}{9}(x - 3)^2 - 1$

e) $f(x) = -\dfrac{2}{9}x^2 + \dfrac{4}{3}x - 3$



12. B **13.**

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